Structural Change, Land Use and Urban Expansion*

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February 14, 2025

Abstract

How do cities grow in the process of structural transformation? To answer this question, we develop a multi-sector spatial equilibrium model with endogenous land use: land is used either for agriculture or housing. Urban land, densely populated due to commuting frictions, expands out of agricultural land. With low productivity and high subsistence needs, farmland is expensive, households cannot afford large homes and cities are very dense. Increasing productivity reallocates factors away from agriculture, freeing up land for urban expansion and limiting the increase in land values despite higher income and urban population. With the area of cities growing faster than urban population, urban density can persistently decline, as in the data over a long period. The quantitative evaluation calibrated to historical data assembled for France over 180 years explains a large fraction of the joint evolution of urban areas, population density and land values across time and space.

Keywords: Structural Change, Land Use, Productivity Growth, Urban Density.
JEL-codes: O41, R14, O11

*We would like to thank Zsófia Bárány, Thomas Chaney, Pierre-Philippe Combes, Cécile Gaubert, Emeric Henry and our colleagues as well as numerous seminar and conference participants for comments and insights. We thank Alberto Nasi and Carla Guerra Tomazini for research assistance. We gratefully acknowledge funding under the Banque de France-SciencesPo Partnership. Marc Teignier acknowledges financial support from the Spanish Ministry of Science, Innovation and Universities, grant PID2022-139468NB-I00, and AGAUR-Generalitat de Catalunya, grant 2021SGR00862. Numerical computations were partly performed on the S-CAPAD/DANTE platform, IPGP, France.

1 Introduction

Since the early years of the industrial revolution, population massively migrated from rural areas towards cities. This widespread phenomenon of urbanization went together with the reallocation of workers away from the agricultural sector towards manufacturing and service sectors—a phenomenon of structural change. How do cities grow when these well-known phenomena occur? Cities can become denser for a given area—growth at the intensive margin. They can also become larger in surface to accommodate more workers—via growth at the extensive margin. Over a long period, cities have been growing essentially in area, at such a fast speed that their average density has been falling. In other words, over time, cities expanded faster in area than in population. We precisely document this stylized fact for France since 1870 but it is also documented on a global scale in Angel et al. (2010) for the recent period. In France, the population of the main cities has been multiplied by almost 4 since 1870, while their area increased by a factor of 30: the average urban density has thus been divided by a substantial factor of about 8. This paper shows that this persistent decline in density, despite the process of urbanization, is well explained by conventional theories of structural change with non-homothetic preferences and augmented with endogenous land use—whereby land can be used for agriculture or urban housing.

A crucial insight of our theory is to consider that the value of agricultural land at the urban fringe determines the opportunity cost of expanding the area of cities for housing purposes. With low agricultural productivity, agricultural goods and farmland are expensive. High agricultural land values make cities initially small in area and very dense as households cannot afford large homes—a manifestation of the 'food problem' (Schultz (1953)). With structural change driven by rising productivity, workers move away from rural areas towards cities, freeing up agricultural land. As the land value at the urban fringe falls relative to income and richer households start being able to buy larger homes, cities expand in area at a fast rate. Together with the reallocation of workers across sectors, reallocation of land use occurs—from agricultural use to urban use. We document that for France, since 1840, about 15% of French land has been converted away from agricultural use. Our theory can account not only for the reallocation of factors away from agriculture but also for the faster growth of cities in area relative to population and the corresponding decline in average urban density—providing a novel mechanism explaining urban sprawl and suburbanization. This complements the traditional Urban Economics view that cities have sprawled following improvements in commuting technologies, which have allowed households to live further away from their workplace (see references in Glaeser and Kahn (2004), Heblich et al. (2020), Redding (2021)).

Our framework also provides novel predictions regarding the historical evolution of land values, which are in line with the evidence in Piketty and Zucman (2014). The value of farmland as a share of income, initially high due to subsistence needs, falls over time with structural change, while the value of urban land rises significantly. Moreover, despite rising housing demand, the fast expansion of cities at the extensive margin due to structural change initially limits the increase in urban land rents and housing prices. When the reallocation of workers and land out of agriculture slows down, the value

of land must adjust to prevent further expansion of cities with rising workers' incomes and housing demand. Land values start to increase at a faster rate. Our theory thus predicts relatively flat land and housing values for decades before shooting up—a prediction which resembles very much the data for France and most advanced economies as best illustrated in Knoll et al. (2017). Therefore, our theory provides novel insights on the joint evolution of the density of cities and land values along the process of economic development. It also helps understanding how the structure of cities, e.g. their urban extent and density, evolves with the process of structural transformation, shedding new light on the origins of urban sprawl.

The contribution of our paper is threefold. First, we document new stylized facts on land use and urban expansion for France since the mid-nineteenth century. In particular, using historical maps and satellite data for the more recent period, we document the historical decline of the density of French cities. Between 1870 and 1950, the average density was divided by about 3 and again by about 2.5 until 1975—the thirty years post-World War II being characterized in France by faster structural change and *rural exodus* (Mendras (1970), Bairoch (1989), Toutain (1993)). Together with the slowdown of structural change in the more recent decades, average urban density did not fall much since. Using novel cross-sectional data on local farmland values, we also show that, in recent times, cities surrounded by more expensive farmland are denser—confirming that the opportunity cost of building at the urban fringe matters for urban sprawl. These novel facts, together with the historical evolution of urban and agricultural land values in France, motivate our theory.

The second contribution is to develop a spatial general equilibrium model of structural change with endogenous land use and multiple cities/regions differing in their productivities. The production side features three sectors: rural, urban and housing. The rural (urban) sector produces agricultural (non-agricultural) tradable goods, the production of the agricultural good being more land intensive. The housing sector produces location-specific housing units using the urban good and land in the process. Land is in fixed supply and land use is rivalrous: land is either used for agriculture or for housing. Following the traditional monocentric model (Alonso et al. (1964), Muth (1969), Mills (1967)), urban land use (cities) emerges endogenously around given city centers due to commuting costs for workers: urban land is more densely populated than rural land and the urban fringe corresponds to the longest commute of a worker producing urban goods. Due to commuting frictions, urban workers are also compensated with a higher wage than rural workers. Importantly, the rental price of land at the fringe of each city must be equalized across potential usages—the marginal productivity of land in the rural sector determining the opportunity cost of expanding further urban land. The last important components of our theory are the drivers of structural change. Structural change is driven by the combination of non-homothetic CES preferences on the demand side, particularly a subsistence consumption for the rural good, and increasing productivity on the supply side. These ingredients generate transitory dynamics with rising productivity in agriculture at the heart of our story: in the old times, due to low agricultural productivity, land is scarce with high values of farmland with respect to income. Moreover, households devote a large fraction of their resources to feed themselves and cannot afford large homes. Few urban workers are concentrated on a small area and urban land is

highly densely populated. Later on, with agricultural development, farmland is getting less valuable, accommodating rising demand for housing of more numerous urban workers. The city sprawls and average urban density might fall through two channels: the fall in the rental price of farmland (relative to income) at the urban fringe and the increasing share of spending towards housing. Note that this decline in urban density can occur even without improvements in commuting technology. Building upon LeRoy and Sonstelie (1983) and DeSalvo and Huq (1996), we account for the latter, more standard, mechanism by parametrizing a model of commuting mode choice, where individuals optimally choose faster commuting modes to live further away from the city center when urban wages increase. Thus, although the mechanisms are entirely different, both urban *and* rural productivity growth can lead to sprawling and suburbanization.

The third contribution is to evaluate the quantitative ability of the spatial equilibrium model to replicate the reallocation of land use and land values in France since 1840. Using data from various historical sources, we measure sectoral factors of production and productivities since 1840 and calibrate the model to fit the process of structural change in France. Historical spatial data on farmland values and urban population discipline the spatial distribution of urban and rural productivity across regions/cities. To account for the use of faster commutes over time, we make use of a tractable parametrization of commuting costs and calibrate the elasticities of commuting speed to urban income and commuting distance using individual commuting data. We show that the model's predictions match relatively well the joint evolution of the urban extent, population density and land value over time and space. More specifically, our framework accounts for most of the decline in average urban density as well as the land value reallocation from rural to urban, and about half of the rise in housing prices since the mid-nineteenth century. Using cross-sectional data on local farmland prices and accounting for possible endogeneity issues, we find that higher farmland values at the urban fringe makes cities relatively denser—a prediction at the heart of our mechanisms. Quantitatively, the elasticity of urban density with respect to the farmland price found in cross-sectional data is in line with its model counterpart. Finally, we disentangle the importance of falling commuting costs relative to our novel mechanism based on structural change in explaining the density of urban settlements. First, we show that without structural change, one cannot match the decline in urban density—emphasizing the key role of improvements in agricultural productivity for urban sprawl. Second, when combined with structural change, the effect of faster commutes is magnified and remains quantitatively crucial to account for the density decline—without faster commutes, the model-predicted density decline over the period would be about 30% of our baseline and short of the data. Third, faster commutes lead to a reallocation of urban workers from the center to the suburbs: central density falls more than average urban density since suburban density increases. To the contrary, structural change leads to the addition of lower and lower density settlements at the urban fringe: suburban density falls more than the average one. While central density did fall since 1870, historical data for Paris shows that it fell less than the average. Our quantitative predictions line up with the Parisian evidence suggesting that both channels—the structural change and the commuting speed channels—are necessary to account for the observed density decline.

Related literature. The paper relates to several strands of literature in macroeconomics and spatial economics. From a macro perspective, it relates to the literature linking productivity changes and land values, starting with Ricardo (1817). This traditional view would imply that a fixed factor such as land should rise in value with economic development (see, among others, Nichols (1970) and Grossman and Steger (2017))—a counterfactual prediction given historical measurement of housing prices and land values (Piketty and Zucman (2014), Knoll et al. (2017), Davis and Heathcote (2007) for related U.S. evidence). An alternative view argues that the rise in land prices can be mitigated by improvements in commuting technologies (Miles and Sefton (2020)). Our approach, in the tradition of the theory of structural change (Herrendorf et al. (2014)), argues that farmland used to be valuable when agricultural productivity was low, but technological improvements can alleviate pressure on land. In a sense, our theory reconciles these different views in a unified spatial framework—adding endogenous land use and a housing sector to the most conventional multi-sector model with non-homothetic preferences (Kongsamut et al. (2001), Gollin et al. (2007), Herrendorf et al. (2013), Boppart (2014), Comin et al. (2021), Alder et al. (2022)). While structural change and urbanization are known to be tightly linked (Lewis (1954)), the spatial dimensions have been rarely investigated. Michaels et al. (2012), Eckert and Peters (2022) and Budí-Ors and Pijoan-Mas (2022) are notable exceptions. The crucial difference to those is the ability of our framework to replicate the evolution of population density within locations, putting emphasis on the internal structure and density of cities, while their focus is on the distribution of population and the sectoral specialization across regions. We also emphasize the implications for land values and show how our framework can generate a sizeable urban-rural wage gap due to commuting frictions—a complementary explanation to the 'agricultural productivity gap' (Gollin et al. (2014)), different from migration costs or selection of migrants (Restuccia et al. (2008), Lagakos and Waugh (2013), Young (2013)).

Our paper also contributes to the literature in spatial economics on urban expansion surveyed in Duranton and Puga (2014, 2015), where commuting costs shape urban density. We add an endogenous sectoral allocation of factors and a general equilibrium structure at the heart of the macro literature. Importantly, the land price at the urban fringe becomes an endogenous object itself affected by structural change. Related work in Brueckner (1990), surveyed in Brueckner and Lall (2015), shows how location-specific land values pin down rural-urban migrations. However, without structural change and endogenous farmland prices, this approach stays quite silent regarding the dynamics of urbanization and land values. In the latter dimension, we contribute to explanations of land values across space (Glaeser et al. (2005), Albouy (2016), Albouy et al. (2018), Combes et al. (2018)). In the French context, we also relate to the historical measurement of land use in Combes et al. (2021b). Lastly, our paper contributes to quantitative spatial economics (Ahlfeldt et al. (2015), Redding and Rossi-Hansberg (2017)) by emphasizing the extensive margin of cities.

The paper is organized as follows. Section 2 provides motivating empirical evidence on land use, land values, urban expansion and population density across space over a long period in France. Section 3 provides a spatial general equilibrium model of land use and structural change. Section 4 evaluates quantitatively the model calibrated to French historical data. Section 5 concludes.

2 Historical Evidence from France

2.1 Land use and Employment in Agriculture

Data. Using various sources described in Appendix A.1, we assemble aggregate data on employment shares in agriculture and agricultural land use in France since 1840. Historical data on land use in agriculture are available roughly every 30 years (or less) until the 1980s and then at higher frequency. They are largely extracted from secondary sources based on the Agricultural Census (Recensement Agricole), and cross-checked with various alternative historical sources (Toutain (1993) among others). Post-1950, data are from the Ministry of Agriculture.

Employment. As all countries going through structural transformation, France exhibits significant reallocation of labor away from agriculture over the period, from about 60% employed in agriculture in 1840 to about 2.5% today (Figure 1). The process of structural change accelerated significantly over the period 1945-1975: in 1945, 36% of the working population are still in agriculture and this number falls below 10% in 1975. In this sense, France is somewhat peculiar relative to the other advanced economies: it is still a largely agrarian economy right after World War II—much more than the U.K. or the U.S.

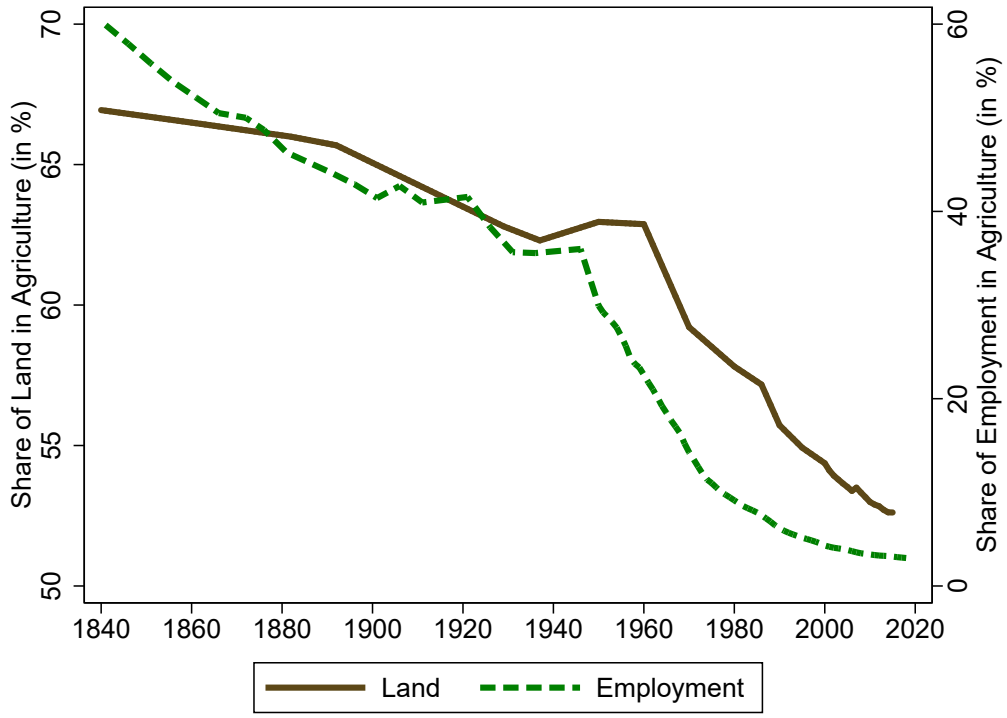


Figure 1: Land use and labor reallocation in France (1840-2015).

Notes: The solid line shows the share of French land used for agriculture (left axis). The dashed line shows the share of workers in the agricultural sector (right axis). *Source*: French Census for Agriculture.

Land use. Although measurement is sometimes difficult for the very early periods, one can confidently argue that, in the aggregate, the share of French land used for agriculture fell significantly

since 1840 (Figure 1).[1] Our preferred estimates are that about two thirds of French land was used for agriculture in 1840. In 2015, this number decreased to 52%. In other words, about 15% of French land use has been reallocated away from agriculture since 1840. While this might not seem quantitatively important, it is substantial from the perspective of urban expansion. 15% of the French territory is actually more than the total amount of land with artificial use in France nowadays, which is about 9% of total land. While it is difficult to assess with certainty what usage former agricultural land has been put to over such a long period, it is likely that a significant fraction of this land has been artificialized, allowing cities to expand. More precise data on land use over the period 1982-2015 show that the surface of artificialized soil increased by about 2 million hectares, or 3.7% of the French territory. This represents roughly 70% of the quantity of land converted away from agriculture over the same period.[2] The measurement of cities area (presented below) provides further compelling evidence that a significant fraction of agricultural land was reallocated towards urban land use.

2.2 Urban Expansion

Data. We use historical maps, aerial photographs and satellite data to measure the area of the main French cities at different dates: 1866 (military maps, e.g. carte d'Etat Major), 1950 (maps and/or photographs), and every ten to fifteen years after 1975 using satellite data from the Global Human Settlement Layer (GHSL) project. One caveat is that we cannot have any area measurement between 1866 and 1950. Data and procedure for the measurement of urban extent across French cities are detailed in Appendix A.2. Measurement of the urban extent using maps in 1866 and 1950 is performed for the 100 most populated cities in the initial period. For a given city, the urban extent ends when the land is not continuously built upon. For the satellite data, it is delimited by grid cells where the fraction of built up land is below 30% and a requirement that cells are connected.[3] By way of example, Figure 2a shows the 1866 map for a medium-size French city, Reims—where one can observe the sharp discontinuity of urban built at the boundary used to delimitate the urban area, even though measurement error at the city-level remains unavoidable (with some farmland included or detached houses inappropriately excluded). On the same scale, Figure 2b shows the same city in 2016 viewed from the sky, with an area of about 20 times larger than in 1866. This figure also clearly

[1] The main issue is the definition of agricultural land. Forests were part of agricultural land in the 19th century but not later. Given their use as natural amenity, we exclude them throughout, even though forest exploitation for wood production is arguably of agricultural nature. The allocation of grazing fields is also not entirely consistent across years before World War II.

[2] Since 1982, data on land use beyond agricultural land use are available on a regular basis from the Enquêtes Teruti and Teruti-Lucas. The rest of agricultural land is to a large extent converted into forests and woods. Forests were accounting for about 18% of French land in 1882 (Agricultural Census) compared to about 30% in 2015 (Enquête Teruti-Lucas)—growing out of agricultural land but also rocky land, moors and sparse vegetation areas.

[3] For maps/photos, the urban fringe is visible by a stark color change between the built and non-built part. See Figures A.6 and A.7 for the precise measurement of the area of Reims using the 1866 and 1950 maps. For satellite data, measurement is not very sensitive to alternative built up thresholds (Appendix A.2.5). Figures A.10 and A.11 illustrate how GHSL data are used to delineate the urban boundaries of Marseille and Bordeaux. We double-check the quality of photo/map measurement in the recent period relative to satellite data measurement. The cross-sectional correlation between both measures is very high. We also cross check our measures with Angel et al. (2010) for Paris and find very similar results. While measurement error when delineating the urban area is unavoidable at the city level, it is less of an issue when averaging across the 100 cities.

(a) Etat Major map of Reims in 1866.

(b) Photograph of Reims in 2016.

Figure 2: The urban expansion of Reims.

Source: *Carte d'Etat Major 1820-1866* and *Photographies aériennes 2016-2020* in https://www.geoportail.gouv.fr, run by the Institut national de l'information géographique et forestière (IGN).

shows how the city is surrounded by agricultural land—a crucial element for our story where urban land expands out of farmland. This feature is not specific to Reims. Recent satellite observations from the Corine Land Cover project show that our sample of cities is surrounded mainly by agricultural land: apart from their coastal part and water bodies, two thirds of land use in the near surroundings of cities is agricultural.[4]

Using Census data, we relate the measured land area occupied by cities to the corresponding population. Data for the first available Census in 1876 are used for the initial period of study. Census data defines population at the municipality level ('commune') and an urban area can incorporate more than one municipality. In 1876, this is not a concern as the main 'commune' of the city is the whole city population. In later periods, one needs to group municipalities into an urban area. Post 1975, GHSL data combines satellite images with Census data on population. This directly provides the population of every grid cell of our measured urban area, circumventing the issue. However, for the 1950 period in between, the different municipalities that are part of our measured areas must be selected. This is done on a case by case basis, looking at the map of each of the 100 largest urban areas. This way, we make sure that the population of the area incorporates all the corresponding municipalities' population.[5]

The area and population of French cities over time. Over time, cities have been increasing much faster in area than in population. Let us give some order of magnitude and describe the average evolution over time for the 100 most populated French cities in 1876. Figure 3 shows the evolution of total area and population of these 100 cities over the period considered—both variables being normalized to 1 to show the increase in size. Since 1870, the area of cities has been multiplied by a factor close to 30 on average. This is a substantial increase. Between 1870 and 1950, the area of cities was roughly multiplied by a factor of 6. Between 1950 and today, the area of cities was

[4]The rest is made of forest/moors and discontinuous urban land (e.g. leisure/transport infrastructure, industrial/commercial sites, ...)—both categories in roughly equal proportions. See Appendix A.2.6 for details.

[5]In 1950, only the largest cities, particularly Paris, are the result of the agglomeration of several 'communes'.

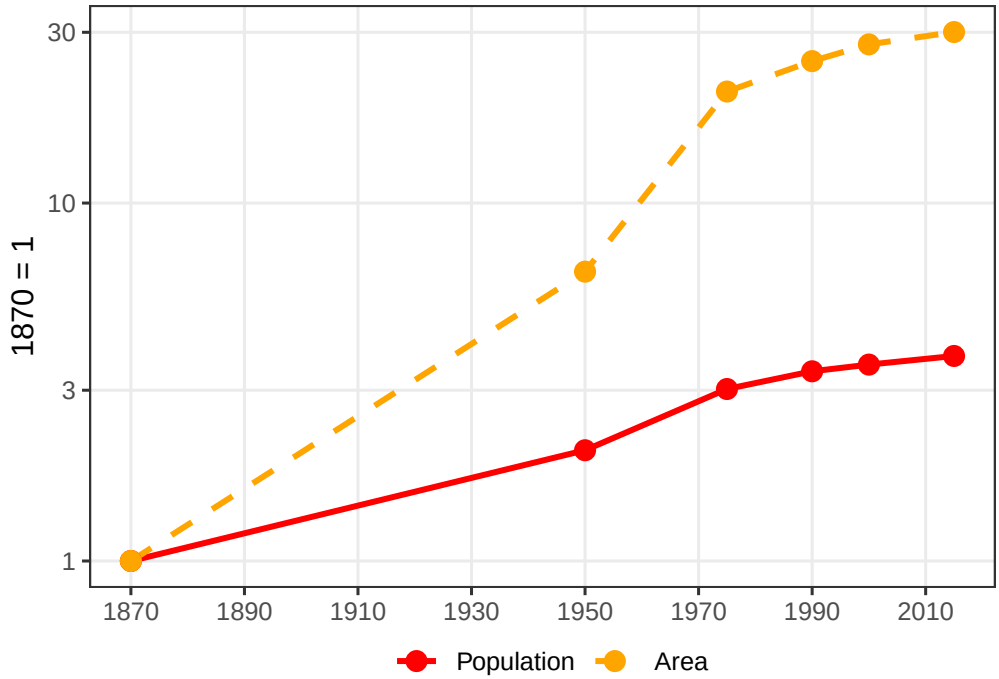
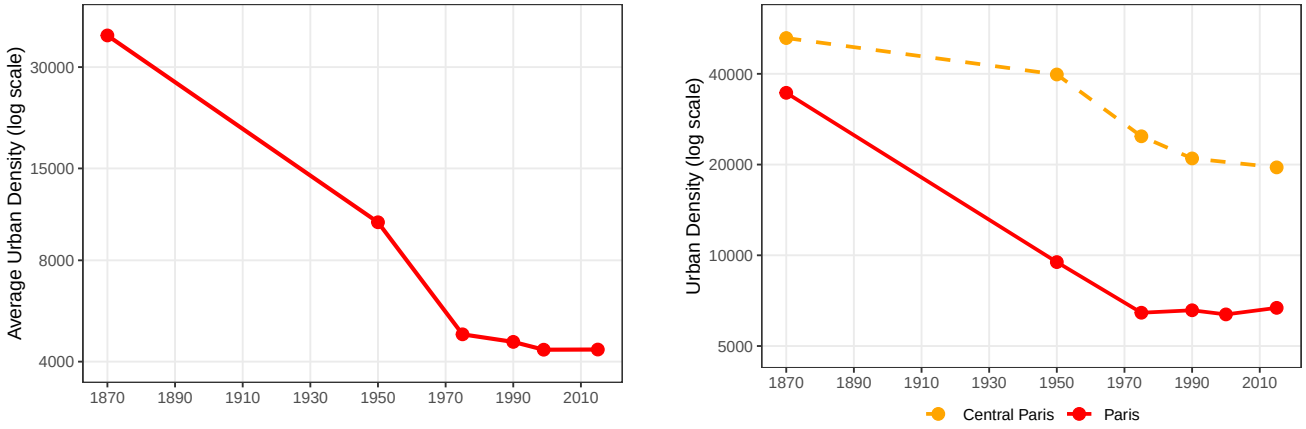


Figure 3: Urban area and population of the 100 largest cities in France (1870-2015).

Notes: The dashed line shows the total urban area of the 100 cities relative to the initial period (sum of all the urban areas) . The bottom solid line shows the total population relative to the initial period in the same cities. Both area and population are normalized to unity in the initial period. *Source*: Etat major, IGN, GHSL and Census.



(a) The decline in average urban density.

(b) The decline in average and central density in Paris.

Figure 4: The historical decline in urban density.

Notes: Left panel: the solid line shows the urban density averaged across the top 100 French cities (weighted average with 1975 population weights). Right panel: the solid line shows the average urban density in Paris; the dashed line the density in Central Paris (districts 1 to 6). *Source*: Etat major, IGN, GHSL and Census.

multiplied again by a factor of 5 on average—the fastest rate of increase being observed over the period 1950-1975. For comparison, the population of these cities has been multiplied by a factor close to 4 since 1870.[6] As urban area increased at a much faster rate than urban population, the average urban density significantly declined over the period.

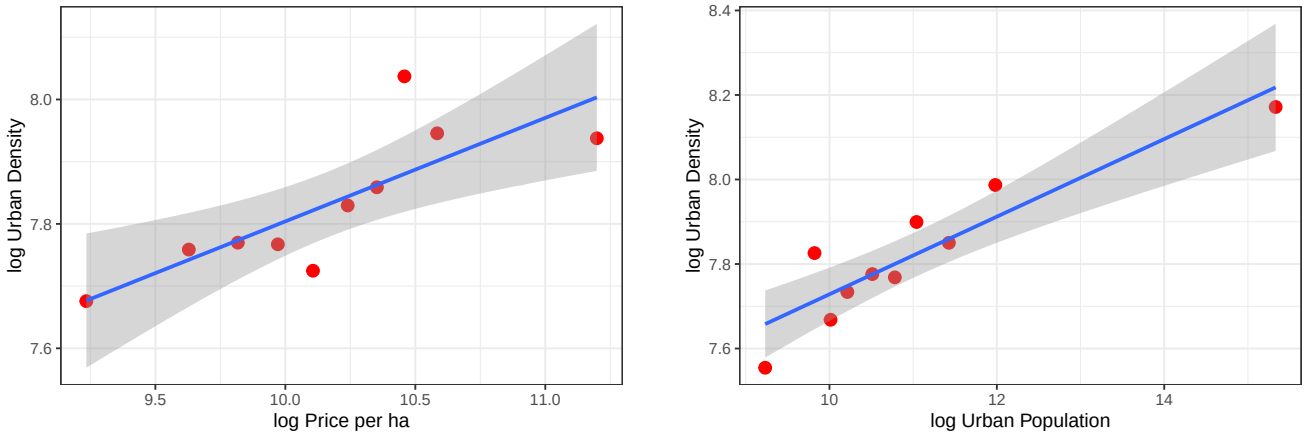
The density of French cities over time. Using population and area of cities at the different dates, one can measure the evolution of urban densities across the different cities over 150 years. While in the cross-section larger cities are denser, the density of French cities declined over time—area expanding at a faster rate than population. This is shown in Figure 4a for the population-weighted average of density across the 100 largest French cities. The average urban density fell massively over the period: it has been divided by a factor of roughly 8. Urban density fell at the fastest rate over the period 1950-1975 and barely falls thereafter. Thus, urban density fell the most over the period when people massively left rural areas and the employment share in agriculture fell the most. The later slowdown of the decline in density coincides with the slowdown in the rate of structural change.

Ideally, one would like to explore how density evolved in different locations of a city (within-city variations). This would provide information on whether density fell in the central locations or in the outskirts of the city. Unfortunately, for most cities we are not able to differentiate the central density to the suburban one as most cities expand the area of their main historical 'commune', particularly so over the period 1870-1950. Thus, we cannot measure the historical population in different parts of a city. However, it can be done for Paris which is divided into several districts. Figure 4b shows the evolution of the density of Central Paris relative to the average urban density of the metropolitan area: the central density of Paris did fall over time but significantly less than the average density of the city. This suggests that the decline in average urban density is not only due to a reallocation of urban residents away from dense centers but also due to the addition of less and less dense suburban areas at the city fringe over time.

Urban density across French cities in recent times. Using satellite data available in the recent period, we build a larger sample of 200 cities for which we measure population and area in the recent years—adding the largest cities in population in 1975 that are not in the initial sample. While the primary focus is to describe the evolution of urban density over long period, we provide insights on a novel determinant of urban density in the cross-section: the price of farmland at the urban fringe. To do so, we use data from the Ministry of Agriculture on the average market transaction prices of arable land (per ha) ('Prix des terres agricoles, terres labourables, libres) at the level of a 'Petite Région Agricole (PRA)'—with more than 700 PRAs in France, this provides a fairly local farmland price surrounding each city (see details in Appendix A.3). Averaging density across cities within each decile bin of farmland prices in 2000, the binned scatter plot in Figure 5a shows that urban density is significantly higher in cities surrounded by more expensive, arguably more productive, farmland.[7] Despite possible endogeneity issues treated in Section 4.4, this preliminary evidence suggests an

[6] French population was multiplied by a bit less than 2 over the entire period. Due to the reallocation of people way from rural areas towards cities, we get roughly a factor 4 over the period.

[7] Results are similar in 2015.



(a) Urban Density and Farmland Price.

(b) Urban Density and Population.

Figure 5: Urban density across French cities in 2000.

Notes: Binned scatter plots. Left-panel: urban density is averaged within each decile bin of farmland prices. Right panel: urban density is averaged within each decile bin of urban population. Urban area, population and density from GHSL data, local farmland prices from the Ministry of Agriculture.

important novel fact at the heart of our story: a lower opportunity cost of expanding cities at their fringe increases urban sprawl. For comparison, we also show the link between urban population and density—more populated cities being denser (see binned scatter plot of Figure 5b, where urban density in averaged within each decile bin of population in 2000). This suggests a quantitatively meaningful effect of farmland prices on urban density: increasing farmland prices around cities from the first to the last decile corresponds to a density increase by about a third, an effect similar in magnitude to an increase in urban population from about 25,000 (3rd decile) to 150,000 (9th decile). Lastly, note that this latter well-known fact linking urban population and density stands in contrast with the decline of urban density in the time-series when cities are getting larger.

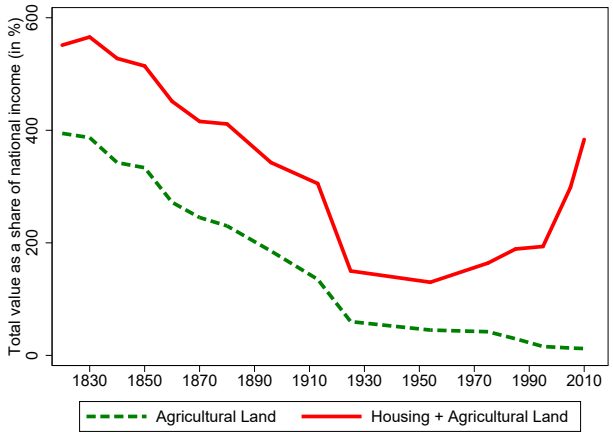
2.3 Land values

Data. Data on land and housing values (over income) for France over a long period can be found in Piketty and Zucman (2014). Historical data for the real housing price index for France are provided in Knoll et al. (2017).

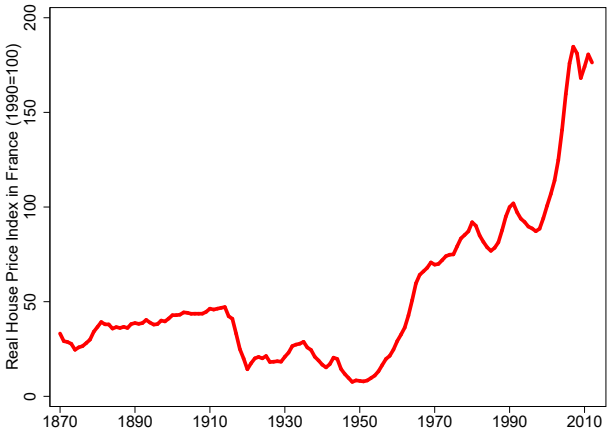
Historical evolution. Figure 6a describes the evolution of the aggregate value of French land over income since 1820. The fall in the value of housing and land wealth (as a share of income) in the pre-World War II period is essentially driven by a declining value of farmland. While farmland was expensive relative to income in the nineteenth century, today it is relatively cheap. This is confirmed by data on average farmland prices: since 1850, the average value of an agricultural field (per unit of land) as a share of per capita income has been divided by a factor of 15 in France. This fact is at the heart of our story: structural change puts downward pressure on farmland values—allowing cities to expand at a fast rate. As a consequence, there is an important reallocation of land values across

usage, from agricultural land towards housing (or urban) land. While the value of agricultural land accounted for more than 70% of housing and land wealth in 1820, it accounts for only 3% in 2010. Lastly, despite the falling value of farmland as a share of income, the total value of housing and land wealth (as a share of income) grows at an increasing rate after 1950.

This steep increase, arguably driven by the increasing value of urban land where most of the population is concentrated, echoes the findings of Knoll et al. (2017).[8] They show that for developed countries, including France, housing prices have been quite stable until the 1950s before rising at an increasing pace—a *hockey-stick* shape of housing prices as shown in Figure 6b.



(a) Agricultural Land and Housing Wealth.



(b) Real House Price Index.

Figure 6: Land and Housing Values in France.

Notes: The left plot shows agricultural wealth as a share of French national income in % (dashed) and the sum of agricultural and housing wealth as a share of national income in % (solid). The right plot shows the housing price index deflated by the CPI. Data are from Piketty and Zucman (2014) (panel 6a) and Knoll et al. (2017) (panel 6b).

To sum-up, our historical data shows a set of salient facts over the last 180 years: beyond the well-known reallocation of labor away from agriculture, land has been reallocated away from agricultural use. Migrations away from the rural areas were accompanied with urban expansion both in area and population. However, given that urban area grew at a significantly faster pace than urban population, the average urban density massively declined over the period, particularly so in the decades following World War II. Together with this process of structural change, the value of farmland as a share of income shrank significantly to the benefit of non-agricultural (urban) land.

These stylized facts motivate our subsequent theoretical analysis. We introduce a spatial dimension together with endogenous land use to the standard theory of structural change with non-homothetic preferences to jointly study agricultural decline, urban sprawl and the spatial reallocation of land values.

[8] Bonnet et al. (2019) show that this increase in the price of housing is largely driven by the price of land and not by the capital and structure component.

3 A Baseline Theory

We present the baseline spatial equilibrium model, describing the environment, deriving equilibrium conditions and defining the equilibrium formally.

3.1 Environment Description

We consider an economy producing an urban good (u) and a rural good (r) at a given date. Time subscripts are omitted for convenience. The urban good is thought of as a composite of manufacturing goods and services, while the rural good represents the agricultural good. The urban good is also used in the production of housing services. Goods and factor markets are perfectly competitive. Both goods are perfectly tradable. The economy is composed of K different regions indexed by $k \in \{1, ..., K\}$ with different productivities. Labor is perfectly mobile across sectors and regions.

Factor Endowments. The economy is endowed with land and a continuum of ex-ante identical workers, both in fixed supply. Each worker is endowed with one unit of labor and we denote by L the total population of workers. Each region k is endowed with land of area S. Land can be used to produce the rural good or for residential purposes. The production of the urban good takes place in the city of each region k, denoted city k, while the production of the rural good, being more land intensive, takes place in the rural area of the region. We assume that production of the urban good takes place in only one location in each city, namely location $\ell_k = 0$ of city k, which is similar to the Central Business District (CBD) in a standard urban model. Regions are assumed to be circular of radius $\sqrt{S/\pi}$ and city k is located at the center of its respective region. Workers' residence ℓ_k can lie anywhere in the region and is denoted by its distance ℓ_k from the center of city k due to symmetry.

Technology. The production of the urban good only uses labor as input. In each region k, one unit of labor produces $\theta_{u,k}$ units of the urban good

$$Y_{u,k} = \theta_{u,k} L_{u,k}$$

where $L_{u,k}$ denotes the number of workers in the urban sector of region k.

In each region k, the production of the rural good uses labor and land according to the following constant returns to scale technology,

$$Y_{r,k} = \theta_{r,k} (L_{r,k})^{\alpha} (S_{r,k})^{1-\alpha},$$

where $L_{r,k}$ denotes the number of workers working in the rural sector in region k, $S_{r,k}$ the amount of land used for production and $\theta_{r,k}$ a Hicks-neutral productivity parameter. $0 < \alpha < 1$ is the intensity of labor use in production.

Remark. The important technology assumption is that the rural sector is more land intensive than the urban one, $1 - \alpha > 0$, implying stronger decreasing returns to scale to labor in this sector.

The production of housing space provided by land developers can use more or less intensively the land for residential purposes. In each location ℓ_k of region k, developers supply housing space $H(\ell_k)$ per unit of land with a convex cost, $\frac{H(\ell_k)^{1+1/\epsilon}}{1+1/\epsilon}$ with $\epsilon > 0$, in units of the numeraire urban good.[9]

Preferences. Preferences over urban and rural goods are non-homothetic as in Kongsamut et al. (2001) and Herrendorf et al. (2013) among others. Consider a worker living in a location ℓ_k of region k. Denote $c_r(\ell_k)$ the consumption of rural goods, $c_u(\ell_k)$ the consumption of urban goods (used as numeraire) and $h(\ell_k)$ the consumption of housing. Workers derive utility only from consumption in location ℓ_k, which is defined as

$$C(\ell_k) = \mathcal{C}\left(c_r(\ell_k), c_u(\ell_k)\right)^{1-\gamma} h(\ell_k)^{\gamma}, \tag{1}$$

where the housing preference parameter γ belongs to $(0,1)$ and the consumption composite $\mathcal{C}$ over rural and urban goods is a non-homothetic CES aggregate with substitution elasticity σ,

$$\mathcal{C}\left(c_r(\ell), c_u(\ell)\right) = \left[\nu^{1/\sigma}\left(c_r(\ell) - \underline{c}\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu)^{1/\sigma}\left(c_u(\ell) + s\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

$\underline{c}$ denotes the minimum consumption level for the rural (subsistence) good, $\underline{s}$ stands for the initial endowment of the urban (luxury) good and the preference parameter ν belongs to $(0,1)$. Preferences are Stone-Geary for $\sigma = 1$.

Urban Spatial Structure. Workers face spatial frictions $\tau(\ell_k)$ when commuting to work in the urban sector of city k. A worker residing in location ℓ_k and working in the urban sector earns a wage *net of spatial frictions* equal to $w(\ell_k) = w_{u,k} - \tau(\ell_k)$, with $w_{u,k}$ denoting the urban wage in city k, $\tau(0) = 0$, and $\partial\tau(\ell_k)/\partial\ell_k \geq 0$. The commuting cost $\tau(\ell_k)$ incorporates all spatial frictions which lower disposable income available for consumption when living further away from the location of production. It includes time-costs of commuting as well as the effective spending on transportation.

Since spatial frictions increase with ℓ_k, urban workers locate as close as possible to $\ell_k = 0$. If one denotes $\ell_k = \phi_k$ the furthest away location of an urban worker, ϕ_k is endogenous in our framework and represents the fringe of city k.[10] Workers residing in locations beyond ϕ_k produce the rural good and do not face spatial frictions, as rural workers do not commute.

We use the functional form $\tau(\ell_k) = a \cdot (w_{u,k})^{\xi_w}(\ell_k)^{\xi_\ell}$, $a > 0$, $\xi_w \in (0,1)$ and $\xi_\ell \in (0,1)$, for which we provide in Appendix B.1.3 a micro-foundation through a commuting choice model. This modeling approach helps mapping commuting costs into observables from commuting data, but results do not depend qualitatively on the micro-foundation as long as commuting costs are increasing and concave in the opportunity cost of time and commuting distance. The concavity, $\xi_w \in (0,1)$ and $\xi_\ell \in (0,1)$, arises from the micro-foundation, whereby individuals optimally choose their commuting

[9] The urban good is used as an intermediary input for the production of housing space. Some equivalent formulation holds for a Cobb-Douglas production function of housing (see Combes et al. (2018)).

[10] Regions are assumed large enough in area such that cities do not expand in neighboring regions. S is large enough such that for all cities, $\phi_k < \sqrt{S/\pi}$.

speed depending on their location ℓ_k and opportunity cost of time (wage rate $w_{u,k}$). This is important as it implies that, for a given residential location, the share of resources devoted to commuting falls with rising urban productivity and wages. In equilibrium, this makes individuals willing to live further away in order to enjoy larger homes. This is the channel through which rising urban productivity leads to faster commutes and suburbanization. The micro-foundation of commuting costs also enlightens the calibration as the elasticity of commuting costs to commuting distance (resp. income) is directly tied to the elasticity of commuting speed to commuting distance (resp. income), both of which have data counterparts.

Remarks. The spatial structure calls for a number of important remarks. First, if it were possible for all workers to locate at $\ell_k = 0$, there would be no spatial frictions. Second, one should note that for $\ell_k \leq \phi_k$, land will be used for residential purposes to host urban workers. As a consequence, land available for rural production would also be maximized if all workers could locate at $\ell_k = 0$. This case would correspond to an entirely 'vertical' city, where land use and spatial frictions are irrelevant. We view this extreme case as a standard two-sector model of structural transformation.

3.2 Household Optimization Conditions

We consider ex-ante identical workers simultaneously choosing their consumption expenditures and their location, taking all prices as given.

Budget Constraint and Expenditures. Consumers earn a wage income net of spatial frictions $w(\ell_k)$ in location ℓ_k of region k. Given the spatial structure, $w(\ell_k) = w_{u,k} - \tau(\ell_k)$ for $\ell_k \leq \phi_k$ and $w(\ell_k) = w_{r,k}$ for $\ell > \phi_k$, where $w_{r,k}$ denotes the wage rate in the rural sector of region k. Consumers also earn land rents, r. Land rents are redistributed lump-sum equally and are thus assumed to be independent of location. Defining p as the relative price of the rural good in terms of the numeraire urban good, the budget constraint of a worker in location ℓ_k of region k satisfies

$$pc_r(\ell_k) + c_u(\ell_k) + q(\ell_k)h(\ell_k) = w(\ell_k) + r, \tag{2}$$

with $q(\ell_k)$ the rental price per unit of housing (or housing price) in location ℓ_k of region k.

Maximizing utility (Equation (1)) subject to the budget constraint (Equation (2)) yields the following consumption expenditures,

$$pc_r(\ell_k) = (1-\gamma)\nu \left(\frac{p}{P}\right)^{1-\sigma} (w(\ell_k) + r + \underline{s} - p\underline{c}) + p\underline{c} \tag{3}$$

$$c_u(\ell_k) = (1-\gamma)(1-\nu) \left(\frac{1}{P}\right)^{1-\sigma} (w(\ell_k) + r + \underline{s} - p\underline{c}) - \underline{s} \tag{4}$$

$$q(\ell_k)h(\ell_k) = \gamma(w(\ell_k) + r + \underline{s} - p\underline{c}), \tag{5}$$

with the composite price index of urban and rural goods, $P = \left[\nu p^{1-\sigma} + (1-\nu)\right]^{\frac{1}{1-\sigma}}$. Due to the presence of subsistence needs ($\underline{c} > 0$), individuals reallocate consumption away from the rural good

with rising income, increasing the consumption share of the urban good and housing (income effects). The reallocation of demand towards the urban good is stronger when $\underline{s} > 0$. The elasticity σ parametrizes substitution effects between rural and urban consumption, vanishing for $\sigma = 1$.

Mobility Equations and Sorting. Since the rural and the urban good are perfectly tradable, urban workers in city k, which would all prefer locations closer to $\ell_k = 0$, compete for these locations. Adjustment of housing prices through the price of land makes sure that households remain indifferent across different locations in a given region k. Using Equations (3)-(5), this implies the following mobility equation, where consumption is equalized to $\overline{C}_k$ across locations ℓ_k,

$$\overline{C}_k = C(\ell_k) = \kappa \frac{w(\ell_k) + r + \underline{s} - p\underline{c}}{q(\ell_k)^\gamma}, \tag{6}$$

with κ constant across locations, equal to $((1-\gamma)\nu)^{(1-\gamma)\nu}\left((1-\gamma)(1-\nu)\right)^{(1-\gamma)(1-\nu)}\gamma^\gamma / P^{1-\gamma}$.

Equation (6) implies that $\left(\frac{w(\ell_k)+r+\underline{s}-p\underline{c}}{q(\ell_k)^\gamma}\right)$ is constant across locations in region k. This holds within urban locations ($\ell_k \leq \phi_k$), within (identical) rural locations, as well as when comparing an urban and rural worker. Since workers in the rural sector do not face spatial frictions and live in ex-post identical locations, $\ell_k \geq \phi_k$, the price of housing must be the same across these locations. We denote by $q_{r,k}$ the price of housing in the rural sector of region k, where $q_{r,k} = q(\ell_k \geq \phi_k)$. A worker in the rural sector earns a wage $w_{r,k}$, receives land rents r and faces the same housing price $q_{r,k} = q(\phi_k)$ than an urban worker at the fringe. Therefore, we have

$$w(\phi_k) = w_{r,k} = w_{u,k} - \tau(\phi_k). \tag{7}$$

In other words, the urban worker at the urban fringe must have the same wage net of commuting frictions than a rural worker—commuting frictions generating an urban-rural wage gap. Equation (7) is essential to understand the spatial allocation of workers: higher spatial frictions at the fringe ϕ_k reduce incentives of rural households to move to the city.

Within city locations ($\ell_k \leq \phi_k$), the housing price adjusts such that workers are indifferent across locations of city k. Using Equations (6) and (7), we get a housing rental price gradient:

$$q(\ell_k) = q_{r,k}\left(\frac{w(\ell_k) + r + \underline{s} - p\underline{c}}{w(\phi_k) + r + \underline{s} - p\underline{c}}\right)^{1/\gamma} = q_{r,k}\left(\frac{w(\ell_k) + r + \underline{s} - p\underline{c}}{w_{r,k} + r + \underline{s} - p\underline{c}}\right)^{1/\gamma}, \tag{8}$$

Within city k, $q(\ell_k)$ is falling with ℓ_k to compensate workers living in worse locations. For ℓ_k above ϕ_k, the housing price is constant, equal to $q_{r,k}$. A crucial difference compared to the standard urban model is that the fringe price $q_{r,k}$ is endogenously determined in our general equilibrium model.

Workers can freely move across regions, therefore equalizing the composite consumption $\overline{C}_k$ of the urban and rural worker at the fringe across the different regions. For all regions $k \in \{1, ...K\}$,

$$\overline{C}_k = \overline{C} = \kappa \frac{w_{u,k} - \tau(\phi_k) + r + \underline{s} - p\underline{c}}{(q_{r,k})^\gamma} = \kappa \frac{w_{r,k} + r + \underline{s} - p\underline{c}}{(q_{r,k})^\gamma}. \tag{9}$$

Equations (6) and (9) guarantee that workers are indifferent between locations within and across regions.

3.3 Producers' Optimization Conditions

Goods producers choose the amount of labor, and land for the rural producer, while land developers choose the supply of housing space in each location ℓ_k, to maximize profits, taking all prices as given.

Urban and Rural Factor Payments. Perfect competition ensures that the urban wage in each region $k \in \{1, ...K\}$ is,

$$w_{u,k} = \theta_{u,k}. \tag{10}$$

Rural workers and land are paid their marginal productivities in each region $k \in \{1, ...K\}$,

$$w_{r,k} = \alpha p \theta_{r,k} \left(\frac{S_{r,k}}{L_{r,k}} \right)^{1-\alpha}, \tag{11}$$

$$\rho_{r,k} = (1-\alpha) p \theta_{r,k} \left(\frac{L_{r,k}}{S_{r,k}} \right)^{\alpha}, \tag{12}$$

where $\rho_{r,k}$ is the rental price of land anywhere in the rural sector of region k.

Housing Supply. Profits per unit of land of the developers are in each location ℓ_k of region k,

$$\pi(\ell_k) = q(\ell_k) H(\ell_k) - \frac{H(\ell_k)^{1+1/\epsilon}}{1+1/\epsilon} - \rho(\ell_k),$$

where $\rho(\ell_k)$ is the rental price of a unit of land in location ℓ_k (the land price). Maximizing profits gives the following supply of housing $H(\ell_k)$ in a given location ℓ_k,

$$H(\ell_k) = q(\ell_k)^{\epsilon}, \tag{13}$$

where the parameter ϵ is the price elasticity of housing supply. More convex costs to build intensively on a given plot of land reduces the supply response of housing to prices. Our framework allows to consider location-specific housing supply elasticities $\epsilon(\ell_k)$ as a straightforward extension—housing supply response might be constrained in some locations (natural constraints, regulations, ...).

Residential Land Prices. Lastly, free entry implies zero profits of land developers. This pins down land prices in a given location,

$$\rho(\ell_k) = \frac{q(\ell_k) H(\ell_k)}{1+\epsilon} = \frac{q(\ell_k)^{1+\epsilon}}{1+\epsilon}. \tag{14}$$

Equation (14), together with Equation (8), implies that land prices are also higher in locations closer to the city center, more so if land developers can build more intensively (higher ϵ). And, for locations beyond the fringe ϕ_k of city k, the land price is constant, $\rho_{r,k} = \rho(\ell_k \geq \phi_k)$, as for the housing price

$q_{r,k}$.

Arbitrage across land use implies that the land price in the urban sector, $\rho(\ell_k)$, must in equilibrium be above the marginal productivity of land for production of the rural good (Equation (12)), where the condition holds with equality in the rural part of the region, for $\ell_k \geq \phi_k$,

$$\rho_{r,k} = \frac{q_{r,k}^{1+\epsilon}}{1+\epsilon} = (1-\alpha)p\theta_{r,k}\left(\frac{L_{r,k}}{S_{r,k}}\right)^{\alpha}. \tag{15}$$

Importantly, this equation shows that a fall in the relative price of rural goods and/or a reallocation of workers away from the rural sector lowers the price of urban land at the city fringe.

3.4 Market Clearing Conditions

Housing Market Equilibrium. Using Equations (5) and (8), the demand for housing space per worker $h(\ell_k)$ in each location of city k is increasing with ℓ_k for $\ell_k \leq \phi_k$,

$$h(\ell_k) = \gamma\left(\frac{w(\ell_k)+r+\underline{s}-p\underline{c}}{q(\ell_k)}\right) = \left(\frac{\gamma}{q_{r,k}}\right)(w(\phi_k)+r+\underline{s}-p\underline{c})^{1/\gamma}(w(\ell_k)+r+\underline{s}-p\underline{c})^{1-1/\gamma}. \tag{16}$$

Facing higher housing prices, households closer to the CBD demand less housing space. A lower fringe price $q_{r,k}$ and lower spending for subsistence $p\underline{c}$ increase the demand for housing space in the city. In the rural area, housing demand per rural worker is constant, $h(\ell_k \geq \phi_k) = \gamma\left(\frac{w_{r,k}+r+\underline{s}-p\underline{c}}{q_{r,k}}\right)$.

Consider first locations within city k, $\ell_k \leq \phi_k$. Market clearing for housing in each location implies $H(\ell_k) = D_k(\ell_k)h(\ell_k)$, where $D_k(\ell_k)$ denotes the density (number of urban workers) in location ℓ_k of city k. The density $D_k(\ell_k)$ follows from Equations (13) and (16), hence

$$D_k(\ell_k) = \frac{H(\ell_k)}{h(\ell_k)} = \frac{q(\ell_k)^{1+\epsilon}}{\gamma(w(\ell_k)+r+\underline{s}-p\underline{c})}. \tag{17}$$

Density for $\ell_k \leq \phi_k$ can be rewritten using Equation (8) and Equation (14) as

$$D_k(\ell_k) = \rho_{r,k}\frac{1+\epsilon}{\gamma}(w(\phi_k)+r+\underline{s}-p\underline{c})^{-\frac{1+\epsilon}{\gamma}}(w(\ell_k)+r+\underline{s}-p\underline{c})^{\frac{1+\epsilon}{\gamma}-1}. \tag{18}$$

Importantly, a lower rural land price $\rho_{r,k}$ at the urban fringe of city k lowers density across all urban locations of the city. Integrating density defined in Equation (18) across urban locations of city k gives the total urban population of the city,

$$L_{u,k} = \int_0^{\phi_k} D_k(\ell_k)2\pi\ell_k d\ell_k = \rho_{r,k}\int_0^{\phi_k}\frac{1+\epsilon}{\gamma}(w(\phi_k)+r+\underline{s}-p\underline{c})^{-\frac{1+\epsilon}{\gamma}}(w(\ell_k)+r+\underline{s}-p\underline{c})^{\frac{1+\epsilon}{\gamma}-1}2\pi\ell_k d\ell_k. \tag{19}$$

Equation (19) pins down the city size ϕ_k. It says that if more workers are willing to move to the urban sector, city will have to be bigger in area to host them—ϕ_k is increasing with $L_{u,k}$.

In the rural area of region k, in locations $\ell_k \geq \phi_k$,

$$L_{r,k}\gamma\left(w_{r,k}+r+\underline{s}-p\underline{c}\right)=S_{hr,k}\left(q_{r,k}\right)^{1+\epsilon}=S_{hr,k}(1+\epsilon)\rho_{r,k},$$

where $S_{hr,k}$ is the amount of land demanded in the rural area for residential purposes in region k.

Land and labor market clearing. Land is used for residential or productive purposes. With total land available in fixed supply S in each region k, the land market clears locally in all regions $k \in \{1, ..., K\}$,

$$S_{r,k}+S_{hr,k}+\pi\phi_k^2=S \tag{20}$$

with the demand of land for housing in the rural area of each region $S_{hr,k}$ equal to $\frac{L_{r,k}\gamma\left(w_{r,k}+r+\underline{s}-p\underline{c}\right)}{(1+\epsilon)\rho_{r,k}}$.

The labour market clears globally. The labor market clearing is such that the total population L is located either in the city or in the rural area of a region k,

$$\sum_{k=1}^{K}L_k=\sum_{k=1}^{K}(L_{r,k}+L_{u,k})=L. \tag{21}$$

Aggregate land rents, rL, include the land rents generated both in the city and in the rural area of each region k,

$$rL=\sum_{k=1}^{K}\left(\int_0^{\phi_k}\rho(\ell_k)2\pi\ell_k d\ell_k+\rho_{r,k}\times(S-\pi\phi_k^2)\right), \tag{22}$$

where it is useful to notice that the rental income in the city exceeds the rental income of farmland for the same area due to spatial frictions.

Good markets clearing. A last step consists in clearing the goods market for rural and urban goods. Rural and urban goods markets clear globally. The rural good is only used for consumption. The market clearing condition for rural goods is

$$\sum_{k=1}^{K}C_{r,k}=\sum_{k=1}^{K}Y_{r,k}, \tag{23}$$

where $C_{r,k}=\left(\int_0^{\phi_k}c_{r,k}(\ell_k)D_k(\ell_k)2\pi\ell_k d\ell_k+c_{r,k}(\ell_k\geq\phi_k)L_{r,k}\right)$ denotes the total consumption of rural goods by urban workers (the first term) and rural workers (the second term) of region k.

The urban good market clearing condition is more involved as urban goods are either consumed, used as intermediary inputs to build residential housing (in all locations) or used to pay for commuting costs. As the condition will be verified by Walras law, the expression is relegated to Appendix B.1.7 (Equation (B.16)).

3.5 Equilibrium Definition

For a given set of exogenous parameters, technological parameters $(\theta_{u,k}, \theta_{r,k}, \alpha)$, commuting cost parameters (a, ξ_w, ξ_ℓ) and resulting spatial frictions $\tau(\ell_k)$ at each location $\ell_k \in \mathcal{L}$, housing supply conditions ϵ, and preference parameters, $(\nu, \gamma, \underline{c}, \underline{s}, \sigma)$, the equilibrium is defined as follows:

Definition 1. *In an economy with K regions, an equilibrium is, in each region $k \in \{1, ..., K\}$, a sectoral labor allocation, $(L_{u,k}, L_{r,k})$, a city fringe ϕ_k and rural land used for production $S_{r,k}$, sectoral wages $(w_{u,k}, w_{r,k})$, a rental price of farmland $(\rho_{r,k})$ together with a relative price of rural goods (p) and land rents (r), such that:*

- *Workers are indifferent in their location decisions, within and across regions, Equations (6) and (9).*
- *Factors are paid the marginal productivity in each region $k \in \{1, ..., K\}$, Equations (10)-(12).*
- *The demand for urban residential land (or the city fringe ϕ_k) satisfies Equation (19) in each region $k \in \{1, ..., K\}$.*
- *The land market clears in each region $k \in \{1, ..., K\}$, Equation (20).*
- *The labor market clears globally, Equation (21).*
- *Land rents satisfy Equation (22).*
- *The rural goods market clears globally, Equation (23).*

The main intuition for the equilibrium allocation goes as follows: in each city k, if the urban sector hosts more workers, the area of the city has to be larger (ϕ_k tends to increase with $L_{u,k}$). However, if the city is larger in area, the worker in the further away urban location commutes more, making the urban sector less attractive for workers: a higher ϕ_k reduces the incentives of workers to move from the rural to the urban sector of city k ($L_{u,k}$ tends to decrease with an increasing ϕ_k). Given technology, the combination of these two forces pins down the allocation of workers across sectors in each region, together with the land used for urban residential housing. Across regions, the allocation of workers is largely driven by differences in regional productivities—more productive regions hosting more workers. Since the equilibrium cannot be described analytically, we provide a simple numerical illustration in Appendix B.2.1 to elucidate the main mechanisms through which increasing productivity in both sectors change the population, area and density of cities. This experiment sheds light on data moments that can be used to identify the model's parameters in the quantitative evaluation of Section 4 and allows us to discuss the modelling assumptions which are important for the main model's implications.

3.6 Discussion

Preferences. With sectoral productivity evolutions, structural change is driven by income effects due to non-homotheticities or by substitution effects for $\sigma \neq 1$. Focusing on income effects ($\sigma = 1$),

rural productivity growth combined with subsistence needs for rural goods frees up land and labor for the urban sector to expand ('rural labor push')—the dominant driver of structural change for a large $\underline{c}$ relative to $\underline{s}$. As illustrated by the experiment in Appendix B.2.1, this perspective replicates qualitatively the salient facts described in Section 2 for France regarding the expansion of the urban area, the evolution of urban density and land values. An alternative view would emphasize a rising demand for (luxury) urban goods as income rises ('urban labor pull')—corresponding to a high $\underline{s}$ relative to $\underline{c}$. While such a calibration can generate employment shares broadly in line with the data, it cannot replicate the observed reallocation of land use and the corresponding fall in urban density. For $\underline{s} > \underline{c}$, as income increases, the spending share on housing falls due to a low income elasticity of housing demand: workers are willing to reduce their housing size to consume more of the urban good. The city does not expand much in area to host more numerous urban workers and urban density might not fall. Importantly, the increase of the housing spending share in the data is informative regarding the relative magnitude of $\underline{c}$ and $\underline{s}$—a crucial insight for the joint calibration of these parameters. We investigate the role of substitution effects for $\sigma \neq 1$ in the quantitative evaluation of Section 4. In the context of France, the main insights are delivered when structural change is driven by income effects since agricultural and urban productivity largely increased at a similar rate in France since 1840 (see Figure 7 below).

Rural Technology. An important insight of the theory is the potential role of *rural* productivity growth for urbanization and the reallocation of workers away from the rural sector ('rural labor push') but also to replicate the large decline in urban density, the fall in farmland prices (relative to income) and the reallocation of land rents towards urban areas. The difference in land intensity between sectors and the substitutability between land and labor in rural production are important for these implications. Intuitively, with a rural land intensity closer to the urban one, the farmland price would decrease less (relative to income) with structural change. As the opportunity cost of expanding the city is higher, this limits the rise in urban areas and the decline in urban density. Similarly, with an elasticity of substitution between land and labor in the rural sector above (resp. below) unity, the farmland price would decrease less (resp. more) with the reallocation of labor to the urban sector as investigated in Section 4.6.[11]

Urban Technology and Commuting Costs. Urban production does not use land and is concentrated in the center. Relaxing only the first assumption is unlikely to change the main outcomes for a land intensity significantly smaller in the urban sector. However, with urban production using land, some activities could be reallocated in the suburbs since central land becomes more expensive as the city grows. With further away residents commuting less, urban density could decline even more. While endogenizing firms and workers location remains a difficult task, we partly capture

[11]The rural production technology remains simple to focus on the core mechanisms. A more sophisticated production (with capital and/or factor biased technical change) could weaken or reinforce the results depending on the substitutability between factors and on the impact of technical change on land per worker. However, it is worth noting that, with commuting frictions, efficiency requires to reallocate labor more than land away from agriculture with structural change—leading to a rise in S_r/L_r and a drop in ρ_r (relative to income). Hence, our theory provides a complementary mechanism to technological explanations of the increase in land per worker in agriculture.

these mechanisms in a later extension where we relax the monocentric assumption—assuming that commuting distance does not map one for one with residential distance (Section 4.6). In this latter Section, we also consider congestion and agglomeration forces absent from the baseline theory. Finally, an important assumption implied by the micro-founded commuting choice model is the concavity of commuting costs with respect to distance and urban wage, $\xi_\ell < 1$ and $\xi_w < 1$. While not necessary, these assumptions appear sufficient to guarantee a drop in urban density in numerical experiments, but less concave commuting costs (higher ξ_ℓ or ξ_w) would limit the increase in urban area and the fall in density.[12] In particular, the magnitude of the income elasticity of commuting costs, ξ_w, matters quantitatively for urban sprawl driven by *urban* productivity growth: facing higher urban wages, urban residents have stronger incentives to relocate in the suburbs to enjoy larger homes when commuting costs increase less with income (a lower ξ_w).

Land use and housing regulations. Our theory abstracts from land use and housing regulations, which would distort equilibrium prices and the equilibrium allocation. Stricter land use regulations aimed at preserving the rural area would limit the expansion of urban areas. This would imply higher urban housing prices together with a higher urban density. While such regulations are currently in place in France, they became effective only in the most recent decades. To the contrary, stricter housing regulations limiting the housing supply in some locations would make cities expand more in area and, consequently, decrease urban density. Such regulations are investigated in a reduced-form way in the quantitative evaluation of Section 4, where the housing supply elasticities are assumed to be lower in the central parts of cities than in the suburbs or the rural part of the economy. This is meant to capture that it is cheaper to build closer to the city fringe than in the city center.

4 Quantitative evaluation for France (1840-2015)

This Section evaluates quantitatively the model developed in Section 3 for France since 1840.

4.1 Quantitative set-up

The time sequence starts in 1840 with steps of 10 years until a final period T far away in the future, $t \in \{1840, 1850, ..., T\}$. The model is calibrated using French historical data over the period 1840-2015 from various sources detailed below. The driving forces are sectoral regional productivity changes and aggregate population growth. After the final data point and until period T, productivity growth is assumed to be constant over time, across sectors and across regions, equal to 1%. Population is growing according to forecasts until 2050 and at a constant rate until T.

For quantitative purposes, we extend the model in two directions. First, we consider a dynamic version of the model. Because of free mobility, the model can be solved as a sequence of static equilibria, but we need to pin down the path of the equilibrium real interest rate and compute

[12] Our approach implicitly assumes that commuting time is taken out of working time entirely. Results would be similar in a framework where commuting time also partly reduces leisure time if leisure is valued at the wage rate.

land values beyond rents. This extension considers a logarithmic instantaneous utility and, given a discount factor β, households maximize their lifetime utility with borrowing and lending in a risk-free asset in net-zero supply. In each location and at each date, land values correspond to the discounted sum of future rents

$$\tilde{\rho}_{k,t}(\ell) = \sum_{s=t}^{\infty} \frac{\rho_s(\ell_k)}{(R_s)^{(s-t)}},$$

where the infinite sum is approximated by truncation for a T large enough relative to t, and the real interest rate R is the ratio of marginal utilities between two consecutive periods.

Second, the supply elasticity of housing space, $\epsilon(\ell_k)$, is allowed to depend on the location within city k (as in Baum-Snow and Han (2023)), with $\partial\epsilon(\ell_k)/\partial\ell_k \geq 0$ and common elasticity in the rural area, $\epsilon(\phi_k) = \epsilon_r$. This is meant to capture higher costs to build closer to the center than in the suburbs or the rural part of the economy. Details of the equilibrium under these extensions are relegated to Appendix B.1.

4.2 Parameter values

For computational purposes, we consider $K = 20$ regions/cities selected among the initial set of 100 cities measured in 1870. One region represents the Parisian area and the remaining 19 cities are randomly drawn from the sample of 100 cities to preserve the distribution of city sizes in terms of population.[13] Each region is initially endowed with the same land area S normalized to unity. Data used for the calibration together with model counterparts are detailed in Appendices B.2.2-B.2.3.

As detailed below, few parameters, $\{\alpha, \beta, \epsilon(0), \epsilon_r\}$, are calibrated using values from the literature. Other parameters are disciplined to match data outcomes. The parameters $\{\xi_l, \xi_w\}$ are estimated separately using micro data on commuting. Population growth is set to match aggregate data. All the remaining parameters are jointly determined to minimize the distance between the model's outcomes and a set of specified moments in the data. Details of the minimization procedure for the joint estimation of parameters $\{\nu, \gamma, \underline{c}, \underline{s}, \sigma, a\}$ together with the distribution of sectoral productivities across regions at each date t, $\{\theta_{u,k,t}, \theta_{r,k,t}\}$, are provided in Appendix B.2.4. The parameter values for the baseline simulation are summarized in Table 1 and the main intuitions behind the identification of the model's parameters are provided below.

Rural production function. The land intensity in agriculture is set to 25%, $\alpha = 0.75$ as in Boppart et al. (2023). Rural production in the quantitative model is Cobb-Douglas but we perform sensitivity with respect to the elasticity of substitution between labor and land, as described in Section 4.6.

Rural and urban productivity. The productivity path for each region k in sector $s \in \{u, r\}$,

[13] We use 1870 for population measures. After selecting Paris by default, we compute median population for the remaining cities, and split the sample at this value. Above the median, we use 10 quantiles of city population to create nine bins, where we draw one city from each bin randomly; below the median we sample from all concerned cities 10 times without replacement. This strategy is employed because below the median, cities are very similar in terms of population, hence choosing randomly amongst all (instead of by bins) ensures better mixing of city types.

Parameter	Description	Value
S	Space per region	1.0
L_{1840}	Initial Population per region	1.0
$\theta_{s,1840}$	Initial Agg. Productivity in sector s	1.0
α	Labor Weight in Rural Production	0.75
ω	Land-Labor Elasticity of Substitution	1.0
σ	Elasticity of Substitution Urban and Rural Good	1.01
ν	Preference Weight for Rural Consumption Good	0.022
γ	Utility Weight of Housing	0.301
$\underline{c}$	Rural Consumption Good Subsistence Level	0.68
$\underline{s}$	Initial Urban Good Endowment	0.17
β	Annual Discount Factor	0.96
ϵ_r	Housing Supply Elasticity in rural area	5.0
$\epsilon(0)$	Housing Supply Elasticity at city center	2.0
ξ_l	Elasticity of commuting cost wrt location	0.55
ξ_w	Elasticity of commuting cost wrt urban wage	0.75
a	Commuting Costs Base Parameter	1.69

Table 1: Aggregate Parameter Values.

Notes: Total initial population in the economy is $K \times L_{1840}$. Total space is $K \times S$.

$\theta_{s,k,t}$, is the product of a common (aggregate) component, $\theta_{s,t}$ and a region-specific component, $\theta^k_{s,t}$,

$$\theta_{s,k,t} = \theta_{s,t} \cdot \theta^k_{s,t}, \tag{24}$$

where the region-specific components are normalized such that aggregate sectoral productivity is equal to $\theta_{s,t}$ at all dates.[14] The path for aggregate productivity in both sectors, $\theta_{r,t}$ and $\theta_{u,t}$, is set to match its data counterpart using aggregate French sectoral data on production, employment and agricultural land use since 1840.[15] The estimated path for $\theta_{r,t}$ and $\theta_{u,t}$ (displayed in Figure 7) is in line with the evolution of the standards of living in France since 1840. It is consistent with the conventional view that the nineteenth century is characterized by a slow agricultural productivity growth relative to the recent decades. More specifically, starting the agricultural crisis in the late nineteenth century, technological progress in French agriculture was slow and delayed relative to other countries, before catching up at a fast rate post-World War II (Bairoch (1989)).

Region-specific sectoral productivities, $\theta^k_{s,t}$, are estimated jointly with the parameters $\{\nu, \gamma, \underline{c}, \underline{s}, \sigma, a\}$ in the minimization procedure described in Appendix B.2.4. However, their estimation relies on some targeted cross-sectional moments, namely the relative population of cities and local farmland values. The targeted population of each city is the population of the delineated urban areas measured using Census data in 1876 and 1950 and satellite data in 1975, 1990, 2000 and 2015 (see Section 2.2). The

[14] The weighted mean of $\theta^k_{s,t}$ is normalized to 1, weighting by population in sector s and region k.

[15] 1840 is the first date of observation for agricultural land use necessary to compute the path of rural productivity. Due to the normalization of price indices, $\theta_{r,0}$ and $\theta_{u,0}$ are set equal to unity in 1840. The yearly path of θs in the data is smoothed to remove business cycles fluctuations.

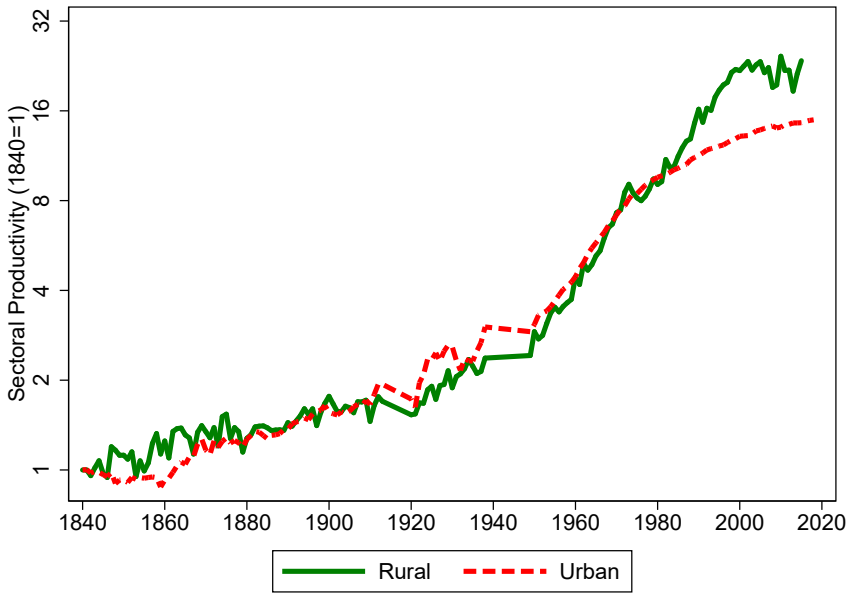


Figure 7: Estimated Aggregate Productivity Series, Rural ($\theta_{r,t}$) and Urban ($\theta_{u,t}$), 1840=1 (1840-2019). Estimation details in Appendix A.1.4.

targeted local farmland values are prices of arable land at the département level in 1892 from the Agricultural Census, and at the level of a département subdivision 'Petite Région Agricole (PRA)' from the Ministry of Agriculture in 1950, 1975, 1990, 2000 and 2015 (see Appendix A.3). For the estimation procedure, the distribution of city populations and local farmland values is kept fixed from 1840 until a first observation date set to 1870, and data are linearly interpolated in between observation dates.[16] Region-specific urban productivities, $\theta^k_{u,t}$, are chosen to match the distribution of population of the 20 different cities. Region-specific rural productivities, $\theta_{r,k,t}$ are estimated to match the distribution of arable land values around each city—where the model-implied price of farmland, $\tilde{\rho}_{r,k,t}$, is the appropriately discounted value of farmland rents located beyond the urban fringe $\phi_{k,t}$ in region k.[17] Specifically, for each rural location ℓ, the value of land is $\tilde{\rho}_{k,t}(\ell) = \sum_{s=t}^{\infty} \frac{\rho_s(\ell_k)}{(R_s)^{(s-t)}}$, as defined in Section 4.1, and the average value of farmland in region k is given by

$$\tilde{\rho}_{r,k,t} = \frac{\int_{\phi_{k,t}}^{\sqrt{S/\pi}} \tilde{\rho}_{k,t}(\ell) 2\pi\ell d\ell}{S - \phi_{k,t}^2 \pi}.$$

Demographics. Aggregate population, L_t, is normalized to the number of regions, K, in the first period and set at each date to match the increase of the French population since 1840 according to

[16] We have three potential dates for the first cross-sectional data point (1866 for the historical map delivering urban areas, 1876 for the population Census, and 1892 for farmland prices). For estimation, we target these initial observations at the unique initial date of 1870.

[17] Within the set of wheat producing regions, our estimates of $\theta_{r,k,t}$ are highly correlated with wheat yields. While the estimation of $\theta_{r,k,t}$ could rely on local agricultural yields, this would require comparing yields for different crops given spatial differences in crop specialization. Data on local farmland values circumvent these issues. See Fiszbein (2022) for the modeling of crop choice across U.S counties.

Census data.[18] Over the period, the French population roughly doubled and the increase in the labor force is of the same magnitude. Going forward, we use the projections for the French population by INSEE until 2050 and a constant growth rate of 0.4% thereafter.

Preferences. Given technology, demographics, and the commuting cost elasticities $\{\xi_l, \xi_w\}$, the preference parameters $\{\nu, \gamma, \underline{c}, \underline{s}, \sigma\}$ are jointly set such that the agricultural employment share and the housing spending share are in line with the data. More precisely, the subsistence needs in agriculture parameter, $\underline{c}$, determines the agricultural employment share in the earlier periods, while the preferences parameter towards the rural good, ν, determines the long-run employment share in agriculture. Similarly, the endowment of urban good, $\underline{s}$, determines the housing spending share for the year 1900 (24% with a 5-year average around 1900)—our initial period of observation regarding consumption expenditures, while the preference parameter towards housing services, γ, determines the housing spending share in recent years (31% in 2010). The parameter governing the elasticity of substitution between the rural and the urban good, σ, determines the impact of relative aggregate sectoral productivity growth on the aggregate sectoral allocation. While aggregate urban and rural productivity increased roughly at the same rate until the 1970s, they moved apart later with faster rural productivity growth (Figure 7). Therefore, for given income-effects parametrized by $\{\underline{c}, \underline{s}\}$, a higher σ implies a slower reallocation of labor away from the rural sector in the recent decades—these later evolutions of the rural employment share pinning down σ. The baseline estimate, $\sigma = 1.01$, suggests that substitution effects are not important to match sectoral employment. However, with little variations of relative sectoral productivity growth, we remain cautious with such an estimate and perform sensitivity analysis with alternative values in Section 4.6.

The last preference parameter, the discount factor β, is irrelevant for the equilibrium allocation given other parameters but pins down the rate of interest and thus matters for the value of land at each date. It is set externally to a standard value of 0.96 on an annual basis, but, within the range of admissible values, results do not depend on the value of β.[19]

Housing supply conditions. Existing estimates of the housing supply elasticities, ϵ, typically vary between 2 and 5, depending on the location as well as on the estimation technique (see, among others, Albouy et al. (2018), Combes et al. (2021a) and Baum-Snow and Han (2023)). Baum-Snow and Han (2023) provides evidence of the *within-city* variation of the housing supply elasticities, ranging from about 2.5 at the central part of the city to about 5 at the fringe of cities. In all regions, we set an elasticity of 2 at location $\ell_k = 0$ and 5 at the fringe and the rural area.[20] For comparison purposes, we perform sensitivity analysis with a constant elasticity of housing supply, $\epsilon = 3$, and we show that

[18] The normalization of the 1840 population together with homogeneous land area S across regions make sure that the land area per person in 1840 is independent of K, equal to $1/S$. Thus, with homogeneous productivities across space, the quantitative model behaves like a one-city model of population normalized to unity in each region.

[19] The minimization procedure detailed in B.2.4 implies computing rural land values around each city but estimates of region-specific productivities aiming at matching relative arable land values barely depend on the value of β.

[20] With Cobb-Douglas production of housing using land and structure, there is a mapping between ϵ and the land share in production. Typical estimates of the land share are between 0.2 and 0.3, corresponding to ϵ between 2 and 4. We assume that $\epsilon(\ell)$ evolves linearly from the central value to the fringe value. Results do not depend on this choice.

the main results do not change (see Section 4.6).

Commuting costs. The elasticities of commuting costs to income, ξ_w, and to distance, ξ_ℓ, are estimated externally using individual level commuting data detailed in Appendix A.5.1. In the model, the elasticity of speed to commuting distance is equal to $1 - \xi_\ell$. We find that this elasticity is precisely estimated within a narrow range around 0.45—depending on the sample used and the controls. Thus, ξ_ℓ is set externally to 0.55.[21]

The elasticity of commuting costs to income ξ_w is tied to the evolution of urban speed when average income increases. More precisely, $(1 - \xi_w)$ is the elasticity of speed to wage income at a given commuting distance. Using the individual commuting data, one can estimate the percentage change in speed over 30 years for a given commuting distance. Over the period 1984-2013, this increase is equal to 11% for an increase in measured aggregate urban productivity of 44%—yielding an estimate for $\xi_w = 1 - \frac{11}{44}$. Thus, ξ_w is set externally to 0.75.

The remaining parameter a is estimated to make the total urban area, $\sum_k \pi\phi_k^2$, represent 17% of rural land in the recent period—the measured artificial land is 17% of the agricultural land in 2010. Results are not very sensitive to a as long as urban land remains a small fraction of available land.

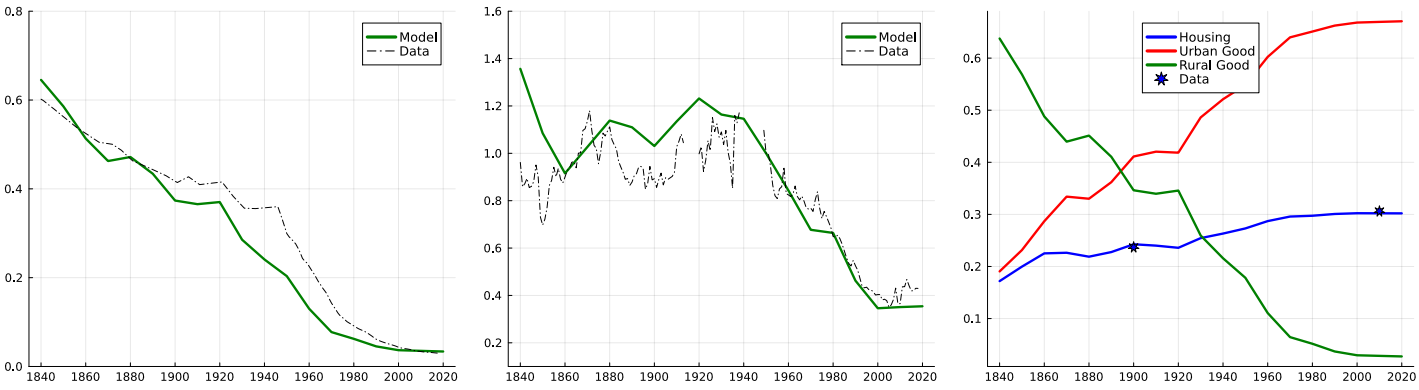
4.3 Results: aggregate outcomes

We first focus on aggregate outcomes over the period 1840-2020 to investigate the ability of the model to reproduce quantitatively the salient facts of Section 2. Model predictions across regions/cities are investigated in a second step. Outcomes are aggregated across regions and compared to aggregate data when available. For urban outcomes, one can interpret the following results as model predictions for the 'average' representative French city.[22]

Structural change. Figure 8 shows that our model is able to account for the patterns of structural change observed in France. As well known in the literature, due to low initial productivity, the (targeted) share of workers needed to produce rural goods is high at the start to satisfy subsistence needs. The demand for rural goods for subsistence makes them initially relatively expensive and households spend a disproportionate share of income on rural goods. Rising rural productivity solves the 'food problem', reallocates labor away from the rural sector and the relative price of rural goods falls. Our model fits the data on the historical evolution of the relative price remarkably well, despite not being targeted (Figure 8b). Moreover, rising income leads to a reallocation of spending away from rural goods towards the urban good and housing services: the spending share on the rural good gradually falls, the share spent on the urban good continuously increases, and so does the (targeted) spending share on housing services, although at a slower speed (Figure 8c). Overall, the spending share patterns are broadly in line with aggregate data if one abstracts from fluctuations in the interwar period.

[21] Commuting data also show that the relationship between speed and commuting distance is very close to log-linear.

[22] Alternatively, these are approximately the outcomes of a city in a region with regional sectoral productivities corresponding to the aggregate ones.



(a) Rural employment share. (b) Relative price of rural good. (c) Spending shares.

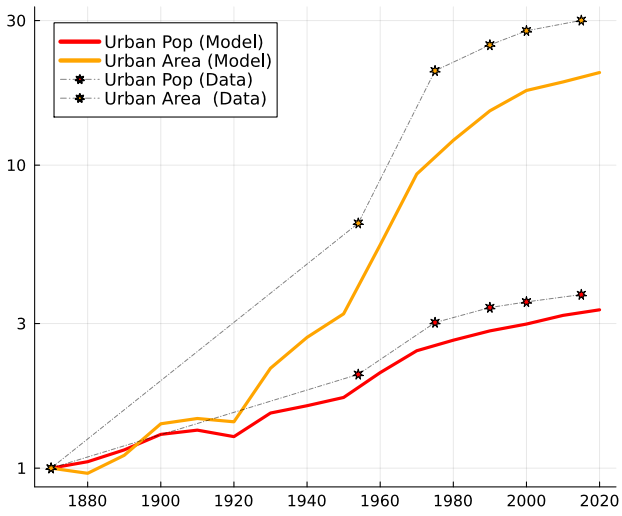
Figure 8: Structural change.

Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. Corresponding data for the employment share, the relative price of rural goods and spending shares are described in Appendix A.1. The relative price is normalized to 1 in 1950.

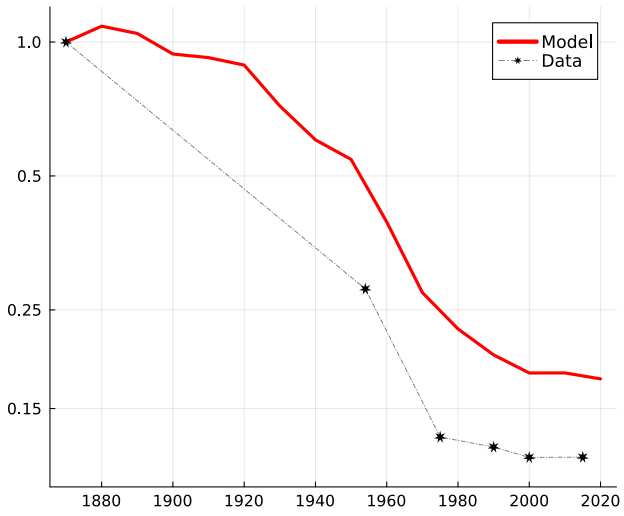
Urban expansion. Figure 9 shows model outcomes that are more specific to our theory with endogenous land use: aggregate urban area (compared to aggregate urban population) and average urban density. For comparison with data on urban expansion, the plots start in 1870—normalizing the value in 1870 to unity. In line with the data, cities expand much faster in area than in population (Figure 9a). While our model does not account for the full observed expansion of the urban area, particularly so until 1950, it explains a very large fraction, despite not being targeted. As a consequence, the model predicts a large fall in average urban density—density is divided by almost 6 since 1870, a bit less than in the data (Figure 9b). The decline in average urban density is the outcome of two different forces—a structural change channel and a commuting cost channel. On the one hand, this is the natural consequence of structural change driven by *rural* productivity growth: higher rural productivity frees up farmland for cities to expand. Combined with less valuable rural goods, this puts downward pressure on farmland prices (relative to income) at the urban fringe. Moreover, as workers spend less on rural goods, they can afford larger homes and spend relatively more on housing. The city expands outwards at a fast rate. On the other hand, changes in commuting costs driven by rising *urban* productivity leads to a reallocation of workers away from the dense center towards the fringe—contributing further to the fall in average urban density. With rising urban income, the share of income devoted to commuting costs falls ($\xi_w < 1$) and workers move towards the suburbs to enjoy larger homes despite a rising opportunity cost of commuting time.[23] Thus, although the mechanisms are entirely different, both rural and urban productivity growth contribute to urban sprawl and falling urban density.

Density within cities. Figure 10 shows the model predictions for density in different locations of the 'average' French city. Figure 10a depicts the evolution of the central density and the density

[23] According to the micro-foundation of commuting costs, this is so because urban workers optimally choose faster commuting modes when moving towards the suburbs, implying $\xi_w < 1$.



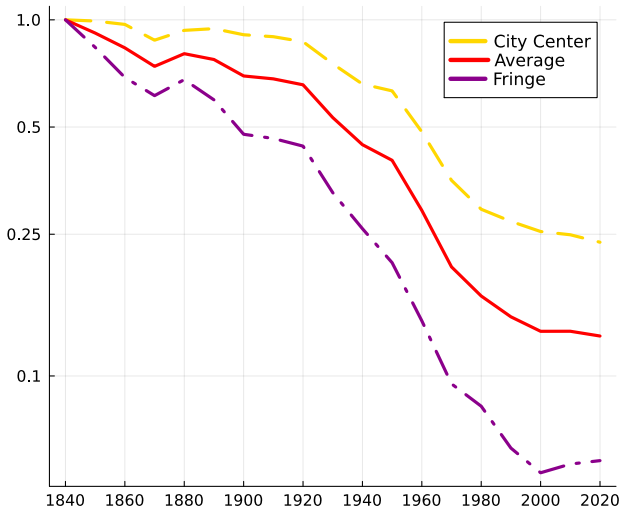
(a) Urban Area and Population (1870=1)



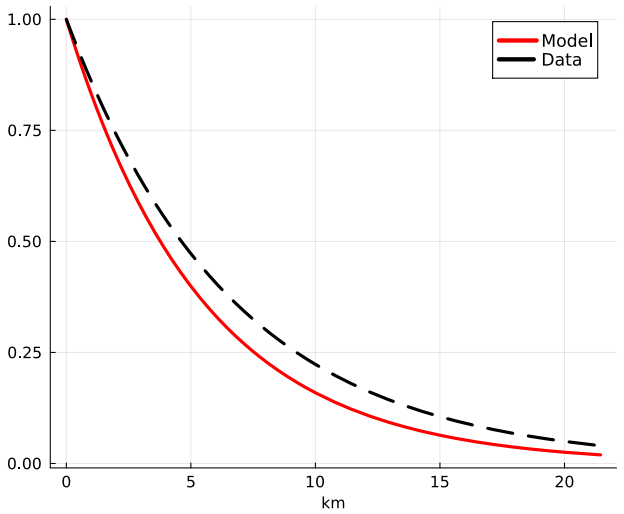
(b) Average urban density (1870=1)

Figure 9: Urban expansion.

Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. Plots start in 1870 for comparison with data. Data and model outcomes are normalized to 1 in 1870 and shown on a log-scale.



(a) Urban density (1840=1).



(b) Density gradient (2020).

Figure 10: Density across space.

Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. Density in different urban locations (left plot) is normalized to 1 in 1840 for readability. Densities are population-weighted averages across cities. Density of the city center is computed on a circle ending at 15% of the initial city radius in 1840. The right panel shows the model implied average exponential decay of urban density in model (year 2000) and data (year 2015). Estimation of model decay is described in detail in Appendix B.2.5, while for data in Appendix A.2.4. Both normalized to 1 at distance 0.

at the fringe of the city (relative to the average), where densities are normalized to 1 in 1840 for readability.[24] The fall in average density is driven both by a fall in central density and a fall in density at the urban fringe. The fall in density at the fringe is the natural consequence of structural change which puts downward pressure on the price of farmland (relative to income). Households can afford larger homes in the suburban parts of the city. Central density also falls because households find it worth to relocate towards the suburbs to enjoy larger homes as they can commute faster when their urban income rises. The former mechanism, more specific to our theory, is crucial to generate a fall in average density that is larger than the fall in the central one—in line with the Parisian data discussed in Section 2. Our model predicts that the overall fall in the central density is about 70% of the fall in the average density—in the ballpark of the estimates for Paris. Lastly, one can measure the density gradient by distance within urban areas, both in the data and in the model in the recent period.. The model predictions are shown in Figure 10b for the 'average' city. The shape of the curve is very close to an exponential (fitted curve) as in the data, and the value of the coefficient of the fitting curve is in the ballpark of the data although slightly higher. Thus, our quantitative model provides a reasonable fit of the data regarding the density of urban settlements within a city and across time.

Commuting speed and the 'agricultural productivity gap'. Using the micro-foundation of commuting costs detailed in Appendix B.1.3, the model generates predictions regarding the evolution of commuting speed across time. Moreover, the marginal urban worker, who has the longest commute, needs to be compensated relative to the rural worker in each region. Our model thus predicts an endogenous urban-rural wage gap, which depends in each region on the city fringe (ϕ_k) and the commuting costs in this furthest away location. These predictions, averaged across regions, are shown in Figure 11.

Over time, our model generates almost a five-fold rise in the average commuting speed (Figure 11a). The endogenous increase in speed is driven by two forces. First, as cities sprawl, urban workers located further away find it worth to commute faster. Second, rising urban income increases the opportunity cost of time and workers choose faster commutes. We collected historical data on the use of different commuting modes for Paris to provide an estimate of the evolution of the average commuting speed in the Parisian urban area (see Appendix A.6 for details). The overall increase in average speed since 1840 predicted by the model is of a similar magnitude than in the Parisian data.[25] Beyond the overall increase, the predictions about the timing line up relatively well with the evolution of commuting speed in the Parisian area. The increase by a factor of about 2 until 1930 reflects the more intensive usage of public transport and their increase in speed over this period (from the initial horse-drawn omnibus to the metro). The later increase, more specifically post-World War II, reflects the increasing car usage.

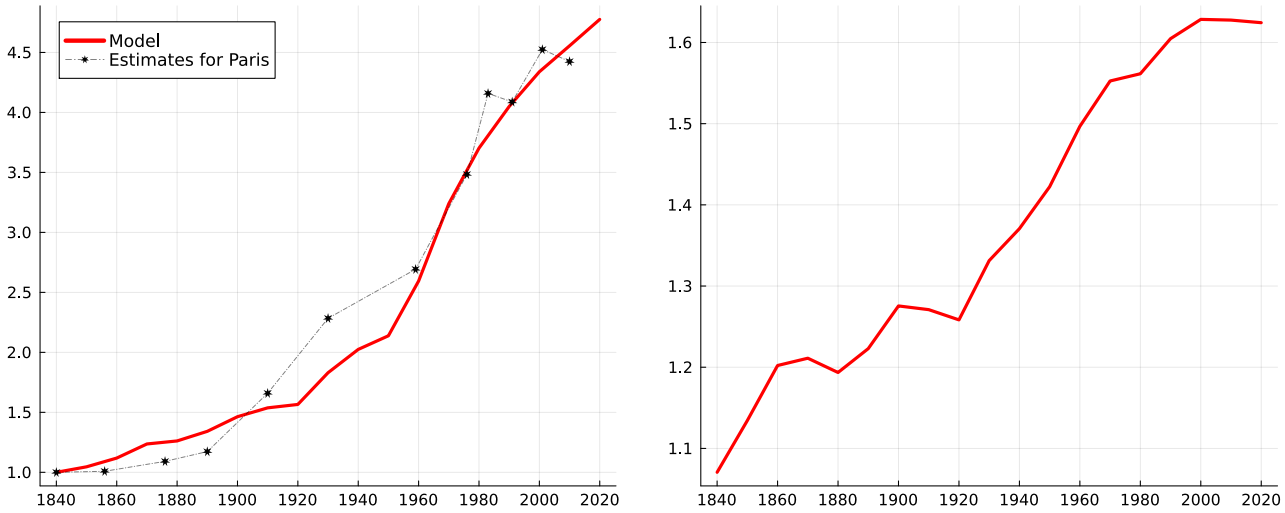
[24] Densities of the 'average' French city are population-weighted average across cities. The fringe of the city center is at 15% of the radius of each city in 1840. Central density is the population-weighted average across cities of the density within this radius.

[25] Miles and Sefton (2020) find a similar increase for the U.K. Historical data are unfortunately not available for the rest of France. The model implied speed in Paris is also very close to the data counterpart.

Following Gollin et al. (2014), Figure 11b shows the 'agricultural productivity gap', averaged across regions. For each region k, the 'agricultural productivity gap' is a monotonic transformation of commuting costs at the fringe of the city—proportional to the urban-rural wage gap, $w_{u,k}/w_{r,k}$. We compute the average raw 'agricultural productivity gap' at a given date as,

$$\text{Raw-APG} = \sum_{k=1}^{K} \left(\frac{L_k}{L}\right) \left(\frac{L_{r,k}/L_{u,k}}{VA_{r,k}/VA_{u,k}}\right),$$

where $\frac{L_k}{L}$ is the population-weight of region k, $L_{s,k}$ and $VA_{s,k}$ denotes the employment and value added in sector s of region k. The value predicted by the model for the recent period, around 1.6, is in line with the values computed by Gollin et al. (2014) for France—lying in between their Raw-APG and Adjusted-APG. Computing the Raw-APG for the entire sample period directly from historical national accounts data, we find that our model falls short of the entire gap, especially for the initial years, but explains a large fraction since 1960.[26] Our quantitative model suggests that spatial frictions combined with location-specific housing can generate urban-rural wage gaps of a significant economic magnitude. It also provides insights on the persistence of fairly large gaps even in developed countries, where labor misallocation is arguably less relevant.



(a) Average urban commuting speed (1840=1).

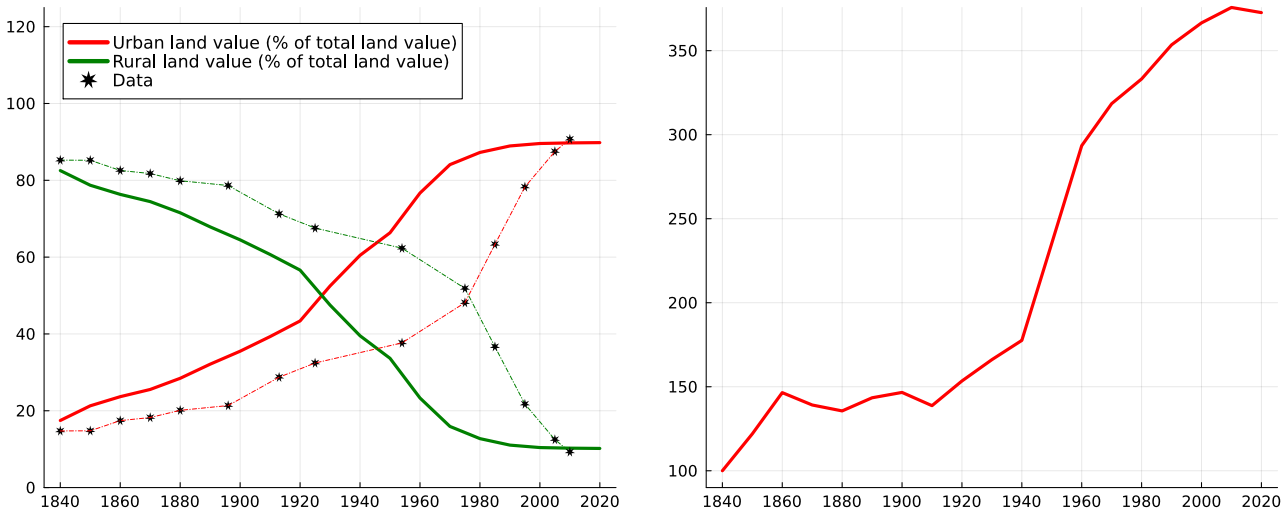
(b) Agricultural productivity gap.

Figure 11: Commuting speed and the 'agricultural productivity gap'.

Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. The average urban commuting speed (left plot) is the density-weighted average of speeds across urban locations (see Appendix B.2.5 for definition, normalization to 1 in 1840). Estimates for Paris are detailed in Appendix A.6. The 'agricultural productivity gap' (right plot) is defined as the population-weighted average across regions of $\frac{L_{r,k}/L_{u,k}}{VA_{r,k}/VA_{u,k}}$.

Land values and housing prices. Figure 12 shows the model predictions for land values and

[26] Using wage data, Sicsic (1992) provides estimates of the urban-rural wage gap in France over the period 1852-1911. Like in the U.K., he finds a significant increase of the gap over the period, in line with our predictions.



(a) Urban versus rural land wealth.

(b) Real Housing Price Index (1840=100).

Figure 12: Land values and housing price.

Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. Land and housing values are computed as the discounted sum of future land rents in each location. Corresponding data (dashed) are based on Piketty and Zucman (2014). The real housing price index averages the purchasing housing prices across locations (deflated using a model implied GDP-deflator). Details on the computation are provided in Appendix B.2.5.

housing prices. Figure 12a shows the reallocation of land value across rural and urban use.[27] Due to structural change, the value of rural land relative to urban land fell dramatically. In the model, while the value of agricultural land constituted more than 80% of the total land value, it is less than 10% nowadays. This is broadly in line with data from Piketty and Zucman (2014) even though our model misses the timing of the reallocation around the time of World War II—arguably due to war destructions.[28] Importantly, the value of urban land (per unit of land) increased faster in the recent decades. This mirrors the evolution of the housing price index since 1840 (Figure 12b), whose shape reminds of the hockey-stick shown in Figure 6b. The model generates about half of the increase in housing prices described in Knoll et al. (2017) post-World War II. Quantitatively, the model misses the very steep increase in the 2000s, most likely due to factors outside the model such as the large decline in interest rates and/or a tightening of land use restrictions.[29]

[27] To compute the urban land value in the data, we multiply the housing wealth by the share of land in housing, whose average is 0.32 in the data for the period 1979-2019.

[28] War destructions arguably delayed the increase in housing wealth (to the post-reconstruction period). This delay has been possibly reinforced by a drop in housing values following the Great Depression and by the rent control imposed in France in between the wars .

[29] France has a planned allocation of land use (agricultural, housing, protected area such as forests) decided at the municipality level. These restrictions are likely to play a larger role at the end of the sample as the law regarding the 'Plans Locaux d'Urbanisme (PLU)', initiated in the year 2000, becomes stricter and more broadly enforced.

4.4 Results: outcomes across regions

While the main purpose of the quantitative model is to reproduce the aggregate facts developed in Section 2, the model with multiple regions/cities provides additional predictions across space. The dispersion across regions of urban and rural productivities, $\{\theta_{u,k,t}, \theta_{r,k,t}\}$, generates dispersion across regions of sectoral employment and wages, of land use and urban density, of urban and rural land values. We focus on the dispersion of urban density and land values, more central in our contribution. We also focus on the implications of the dispersion of rural productivity since a crucial aspect of our story is the role of rural productivity for the expansion and density of cities.

Region-specific productivity changes. Before investigating the model predictions across space, it is important to clarify the response of a given region facing regional productivity changes in sector s, changes in $\theta^k_{s,t}$, as opposed to common (aggregate) productivity changes, i.e. changes in $\theta_{s,t}$.

In response to a local increase in rural productivity $\theta^k_{r,t}$, region k sees its rural sector expand in terms of employment and value added, while city k shrinks in area. Intuitively, a rise in region k's rural productivity leads to higher rural wages and land values in region k. Region k, then, attracts rural workers from other regions, which further increases rural land values there. With higher prices at the urban fringe, urban land and housing prices increase, making city k less attractive. As a consequence, urban area in city k falls and urban density increases.[30] This latter prediction is at the heart of our story: higher land prices at the fringe of cities increase urban density.

It is important to note that the predictions for region k are drastically different when the increase in rural productivity is common across regions (an increase in $\theta_{r,t}$). In this case, the rural sector shrinks and rural land prices drop in *all* regions, since structural change forces operate. As workers move to the urban sector, all cities expand both in area and population, but faster in area: urban density decreases as illustrated in Section 4.3. In other words, for a given change in rural productivity $\theta_{r,k,t}$ in region k, the response is drastically different whether the productivity change is local or common. General equilibrium effects through the relative price of rural goods following a common (aggregate) increase in rural productivity are crucial for the result—a reminiscence of the role of rural productivity for structural change in open versus closed economies (Matsuyama (1992), Gollin (2010), Uy et al. (2013), Bustos et al. (2016), Teignier (2018) among others).[31]

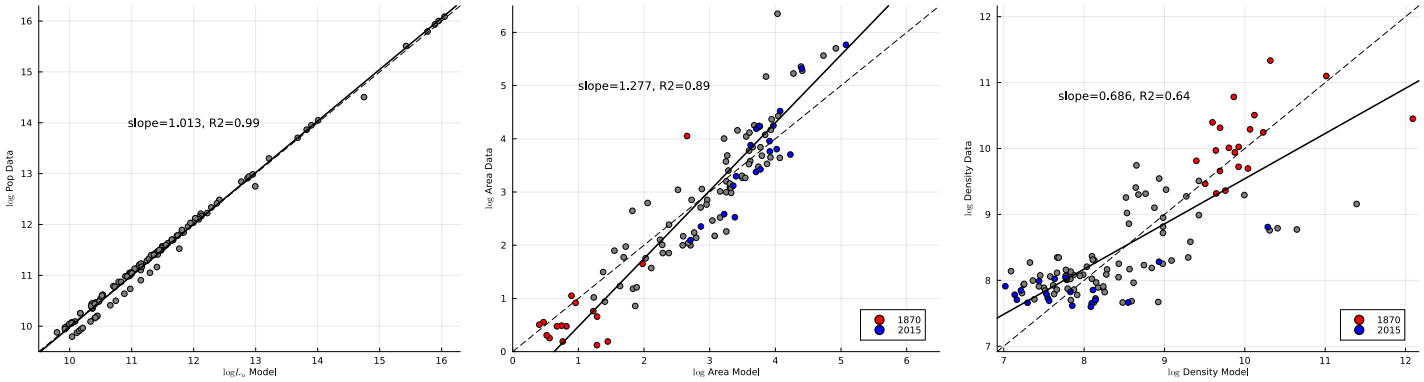
Similarly, a higher region-specific urban productivity, $\theta^k_{u,t}$, significantly increases the size of city k, both in population and area—workers from other cities move towards the relatively more productive city. Due to higher housing prices, city k gets then relatively denser. To the opposite, a common increase in urban productivity, $\theta_{u,t}$, barely increases the population of city k—the same amount of rural workers is needed to feed the urban population. The rise in $\theta_{u,t}$ does, however, lead to a fall in the density of all cities, as urban area increases due to faster commuting modes.

[30] To the opposite, the rural sector in other regions shrinks while their respective cities expand—the effects might be relatively small though if region k accounts for a small share of total employment.

[31] See also Donaldson and Hornbeck (2016) for the role of falling trade costs for regional agricultural specialization.

Thus, again, depending on their local or global nature, productivity changes in a given city k have entirely different implications for urban population and density. While variations in the time-series are arguably dominated by aggregate productivity changes (Section 4.3), region-specific productivity changes might generate very different cross-sectional implications. We now investigate further some of these implications across regions.

City size and urban density. Beyond the targeted distribution of population across cities, the model does a decent job at reproducing the distribution of urban area and average urban density across time and space (see Figure 13). In particular, Figure 13c plots the log of average urban density in a given city against its data counterpart for the dates where it is observed in the data (1870, 1950, 1975, 1990, 2000 and 2015).[32] The model predicts that, over time, for a given city, urban density falls as urban population increases following common (aggregate) productivity changes—in line with the aggregate results. In the cross-section, due to higher housing prices, more populated cities are however denser. Both predictions, over time and in the cross-section, are qualitatively in line with the data discussed in Section 2. Quantitatively, the model does notably better in the time-series than in the cross-section. At a given date, more populated cities are significantly denser in the model than in the data (visible in Figure 13c for the largest and densest cities).[33] Overall, with only productivity differences across regions, our model falls short of explaining the cross-sectional dispersion of urban density, particularly so in the recent period.



(a) Urban Population. (b) Urban Area. (c) Urban Density.

Figure 13: Regional Urban Moments.

Notes: We plot the log of model population/areas/density vs the log of population/areas/density in the data for all observed dates. Dotted 45° line and solid (pooled) regression line of model against data. Variables centered such that the mean in the data across observations matches the model's counterpart. Data and model outcomes are for dates $t \in \{1870, 1950, 1975, 1990, 2000, 2015\}$, with model outcomes interpolated to obtain 1975 and 2015 values. Sample of 20 cities. Outcomes of the baseline simulation where parameters are set to the values of Table 1.

Urban density and rural land values. A second important implication, crucial for our mecha-

[32] We interpolate model outcomes for 1975 and 2015. Model outcomes are defined up to a constant of normalization defining the measurement unit; normalization such that the mean across all observations matches the data counterpart.

[33] The issue is the most severe for Paris. Relaxing the monocentric assumption in Section 4.6 helps to some extent but overall, our model generates an order of magnitude too large Parisian density.

nisms, goes as follows: a relatively higher rural productivity in region k, higher $\theta^k_{r,t}$, increases land prices at the fringe of city k, leading to higher density in city k. Following the evidence in Section 2.2, we investigate the link between average urban density in a given city and its farmland price at the fringe using satellite measures of urban density and the corresponding local price of arable land of the 'Petite Région Agricole'. We perform the following regression in the model and in the data,

$$\log \text{density}_{k,t} = a_t + b \cdot \log \bar{\rho}_{r,k,t} + c \cdot Z_{k,t} + u_{k,t}, \tag{25}$$

where $\text{density}_{i,t}$ is the average urban density of city k, $\bar{\rho}_{r,k,t}$ the farmland price around city k, a_t a time-effect and $Z_{k,t}$ region/city-specific controls. Controlling for aggregate changes through a_t, the model unambiguously predicts $b > 0$, when controlling for region-specific urban productivity, $\theta^k_{u,t}$. In other words, a city in region k should be denser when the value of farmland is higher, holding everything else constant. When turning to the data, two important caveats are in order: measurement issues and endogeneity concerns. For the latter, beyond possible reverse causality, unobservable local characteristics (e.g., land use regulations or local amenities) might simultaneously affect the local price of farmland and urban density. To address these issues, we instrument local farmland prices using département-level data on wheat yields focusing on a sub-sample of cities in départements where wheat is one of the main crops. Given the reduced sample, we use a larger sample of cities, the 200 largest French cities, to preserve statistical power.

	log Urban Density		
	Model	Data (OLS)	Data (IV)
$\log \bar{\rho}_{r,k,t}$	0.371***	0.126***	0.346***
	(0.018)	(0.026)	(0.098)
Controls	$\log w_{u,k,t}$	$\log w_{u,k,t}$	$\log w_{u,k,t}$
Num.Obs.	80	766	314
R2	0.994	0.253	0.272
FE: year	X	X	X

Table 2: Urban density and rural land values.

Notes: Results of Regression Eq. 25 in the model and in the data for years $t \in \{1975, 1990, 2000, 2015\}$. Model regressions based on outcomes of the baseline simulation of the quantitative model with a set of $K = 20$ cities. Farmland values in region k, $\bar{\rho}_{r,k,t}$, computed as the discounted sum of future land rents beyond the urban fringe $\phi_{r,k,t}$ in region k. Average urban density, $\text{density}_{k,t}$, is the urban population $L_{u,k,t}$ of city k divided by its area $\pi\phi^2_{k,t}$. Data on local farmland value $\bar{\rho}_{r,k,t}$ is the price of arable land in the Petite Region Agricole (PRA) of city k. Average urban density is measured using GHSL data for a sample of 200 cities. For IV-regressions, local farmland values are instrumented by wheat yields on the restricted sample of cities in départements with wheat as one of the main crops in 2000. Controls are urban wages (in log), $w_{u,k,t}$, in city k in model and data. Std Errors clustered at the département level. Signif. Codes: ***=0.01, **=0.05, *=0.1.

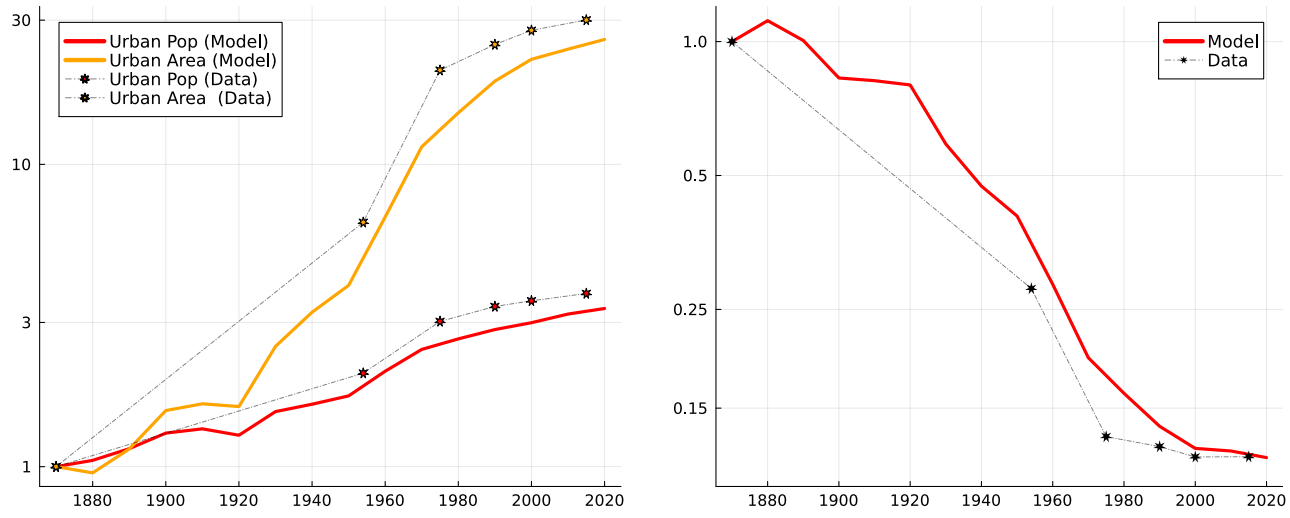
Our baseline IV-estimates using this subsample of cities are shown in Table 2 together with the OLS estimate on the whole sample of 200 cities measured in years 1975, 1990, 2000 and 2015. Results are

striking: cities in locations with higher farmland values are denser. Quantitatively, the IV-estimated elasticity is relatively close to its model's counterpart—a 10% increase in the local farmland value increasing urban density by about 3.5%. Details of the empirical strategy together with sensitivity analysis and robustness checks are relegated to Appendix A.4. Our baseline IV-strategy provides a direct mapping between the primitives of the model (region-specific agricultural productivity) and the data (region-specific wheat yields in départements growing wheat) but at the cost of using essentially cross-sectional variations in yields to instrument farmland prices. To circumvent this issue, we also provide a different IV-strategy which relies on time-series variations and allows to control for local fixed-effects. Levering up on the availability of yields for different crops combined with different crop specialization across French départements, we build shift-share instruments of farmland prices by interacting national changes in yields of each crop with the share of land use for the different crops at an early date. At the cost of weaker instruments due to the limited timespan, this strategy gives estimates of similar magnitude than our baseline cross-sectional identification. Beyond validating the cross-sectional prediction, these results provide more convincing evidence of our mechanisms over time, whereby lower rural land values at the fringe of cities lowers urban density along the process of structural change.

4.5 Counterfactual Experiments

In order to shed further light on the mechanisms at play and discuss the sensitivity of our results to the different elements of the model, we perform counterfactual experiments. These experiments aim at showing how aggregate productivity changes, structural change and the use of faster commutes contribute to urban expansion. However, it is important to note that the structural change and commuting costs channels interact with each other, most notably structural change magnifies the commuting costs channel, and this makes it difficult to account quantitatively for their respective contribution.

Counterfactual with fixed cross-sectional heterogeneity. The baseline estimation combines the effects of aggregate productivity changes and region-specific productivity changes on area and density of cities. To isolate the effect of aggregate changes, at the heart of our mechanisms, we perform a counterfactual fixing cross-sectional heterogeneity to its initial value—leaving all parameters but region-specific ones to their baseline value. While the evolution of the rural employment share, of the relative price of rural goods and of spending shares are barely affected, outcomes regarding urban expansion, more specific to our theory, are quantitatively different from the baseline. Specifically, this counterfactual leads to more urban sprawl and to a larger decline of average urban density compared to our baseline—bringing their evolution closer to the data (see Figure 14). While validating the importance of aggregate productivity changes for the results, this counterfactual with fixed-cross sectional heterogeneity also isolates the importance for the baseline results of the reallocation across regions driven by region-specific productivity changes. It shows that these composition effects across regions matter to some extent in the aggregate for the baseline. They are largely driven by the reallocation of urban workers towards large cities, Paris in particular, whose population grew faster

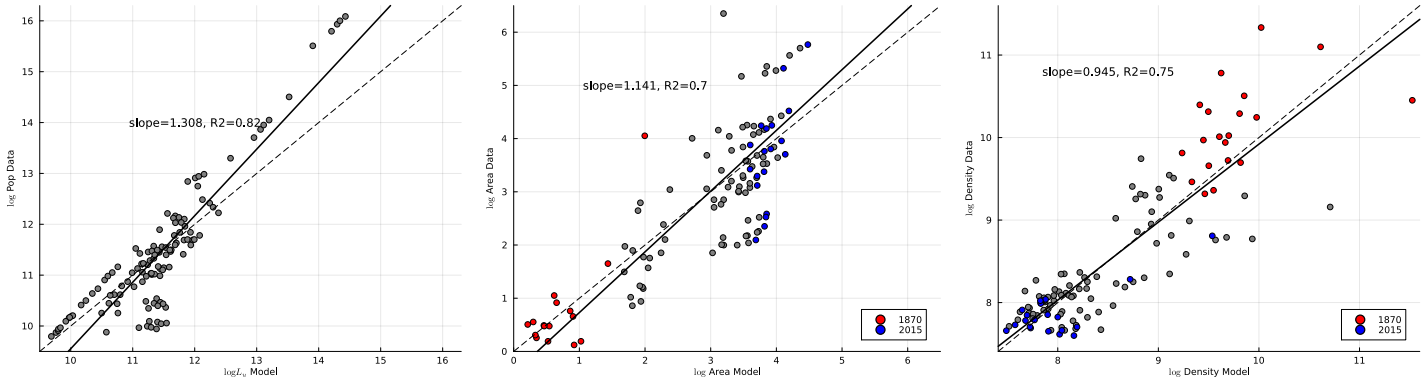


(a) Urban Area and Population (1870=1)

(b) Average urban density (1870=1)

Figure 14: Urban expansion (Fixed Cross-sectional Heterogeneity).

Notes: Regional sectoral productivity differences are constant to the 1870 value. Outcomes of the simulation with fixed cross-sectional heterogeneity where other parameters are set to their baseline values.



(a) Urban Population.

(b) Urban Area.

(c) Urban Density.

Figure 15: Regional Urban Moments (Fixed Cross-sectional Heterogeneity).

Notes: Regional sectoral productivity differences are constant to the 1870 value. We plot the log of model population/areas/density vs the log of population/areas/density in the data for all observed dates. Variables are centered such that the mean in the data across observations matches the model's counterpart. Data and model outcomes are for the dates $t \in \{1870, 1950, 1975, 1990, 2000, 2015\}$, with model outcomes interpolated to obtain 1975 and 2015 values. Sample of 20 cities. Outcomes of the simulation with fixed cross-sectional heterogeneity where other parameters are set to their baseline values.

than smaller cities. As these cities are denser, average (aggregate) urban density falls less in the baseline than in the counterfactual with fixed-cross sectional heterogeneity.

Looking at cross-sectional urban outcomes (Figure 15), this counterfactual cannot, by construction, fit as well the relative population of cities but implies cross-sections of urban density closer to the data. This echoes the limited ability of the baseline estimation to account for the differential evolution of density across cities discussed in Section 4.4. While the model predicts relatively well the density decline in all cities due to aggregate productivity changes, the baseline estimation overstates the increase in density in large cities growing faster relative to smaller ones.

In the next counterfactuals, we aim at disentangling further the mechanisms driving the decline in urban density, most notably the structural change channel tied to improvements in rural productivity to the commuting costs channel.

The role of structural change. How much would have density declined without (or less) structural change? To answer this question, it is useful to shut down the main driver of structural change and perform a counterfactual with lower aggregate rural productivity growth. We perform simulations with an almost stagnating (resp. slowly growing) rural productivity, where the growth rate of θ_r is 4% (resp. 20%) of the baseline at each date.[34] While reducing aggregate rural productivity growth, the urban region-specific components, $\theta^k_{u,t}$, are re-estimated to preserve urban aggregate productivity growth and the distribution of city populations.[35] All other parameters are kept to their baseline values. Results of these simulations are shown in Figure 16 for some variables of interest (aggregated across cities) together with the baseline simulation for comparison. Without or much less structural change, or equivalently with lower improvements of the rural technology, the urban density falls significantly less and might even increase with a sufficiently low rural productivity growth (Figure 16a). Population and urban productivity growth put pressure on land in the rural area to feed an increasingly numerous and richer population. This increases the relative price of rural goods and the price of farmland at the urban fringe (Figure 16c)—preventing the city to expand. Furthermore, facing higher price of rural goods, households reduce their housing spending share to feed themselves, reducing the demand for urban land. These forces tend to make the city much denser than our baseline—more so at the urban fringe due to rising farmland values (Figure 16b). It is also worth emphasizing that population growth, by putting pressure on land, makes agricultural productivity growth even more crucial to generate a sizable expansion in urban area.

This experiment does not say that improvements in commuting technologies do not matter for the expansion in area of cities. However, it makes clear that they matter only when combined with rural productivity growth and structural change. In this counterfactual, urban density might increase

[34] With 4% of the baseline aggregate rural productivity growth rate, the share of rural employment stays roughly the same over the whole period. We refer to this as the no structural change counterfactual.

[35] Although not crucial for the results, re-estimating the region-specific urban productivities preserves aggregate urban productivity and facilitates the numerical solution: otherwise workers are moving massively to Paris due to its faster (baseline) urban productivity growth. With a low rural growth, workers must come from small cities (instead of the rural area), which increases aggregate urban productivity, empties some cities and leads to corner solutions.

despite a significant rise in commuting speed due to rising urban productivity. This is so because higher urban wages make individuals commute faster but the impact on their location decisions is ambiguous: on one side, it increases the opportunity cost of commuting time, attracting people to the center; on the other side, it makes them willing to increase their housing size and relocate to the suburbs. Without structural change, the latter force is muted due to subsistence needs: urban productivity growth and faster commutes have much less of an effect on urban sprawl. The next experiment provides further insights on the quantitative role of commuting costs when combined with structural change.

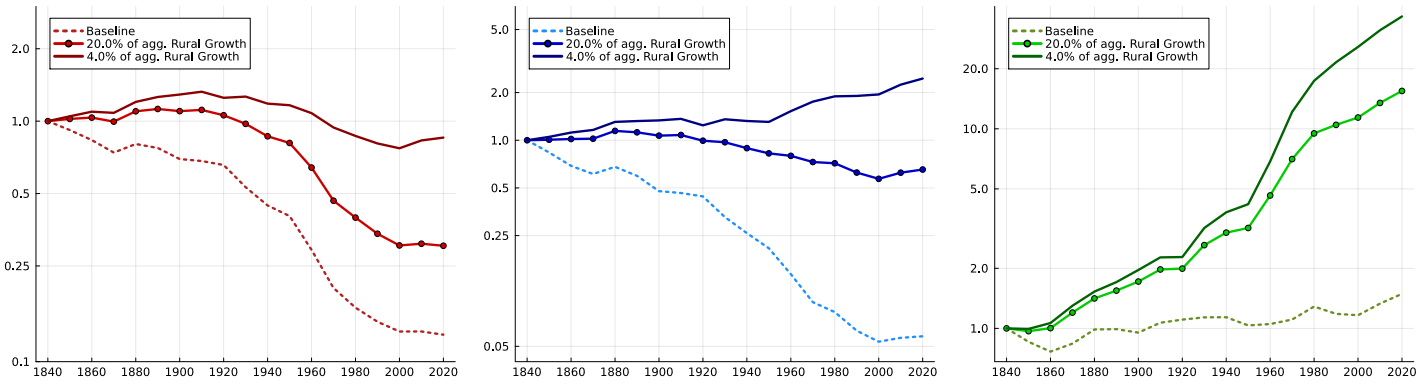
The role of commuting costs. In presence of structural change, how much would have density declined without (or less) increase in commuting speed? To shed light on the quantitative importance of falling commuting costs and rising commuting speed, we set the elasticity of commuting costs to income, ξ_w, to unity, $\tau(\ell_k) = a.w_{u,k}.\ell_k^{\xi_\ell}$.[36] All other parameters are set to their baseline values. In such a calibration without income-effects on commuting, the fraction of wages devoted to commuting in a given location does not fall with rising urban productivity: contrary to our baseline, the speed of commuting does not increase with rising urban wages. When compared to the baseline, this illustrates the quantitative role of the use of faster commutes with rising urban productivity when combined with structural change. Figure 17 shows the results aggregated across cities in this alternative calibration together with the baseline for comparison. Figure 17a makes clear that increasing the elasticity of commuting costs to income severely limits the increase in the average commuting speed over the period. As the cost of faster commutes increases more, urban workers do not relocate away from central locations towards the suburbs of the city as much. This severely limits the sprawl of the city and the fall of the average urban density (Figure 17b)— the counterfactual change in (log) average urban density being 30% of the baseline and about 25% of the data since 1870.[37]

Thus, when combined with rural productivity growth, the use of faster commutes and the corresponding decline in commuting costs (as a share of the urban wage) is quantitatively important to account for the overall decline in urban density—particularly so in central locations. In this alternative experiment, as the urban area expands much less but urban population grows essentially as much due to structural change, urban land values and housing prices increase much more than in the baseline (Figure 17c).[38] This mirrors the role of improvements in commuting modes to limit the increase in urban land values emphasized in Heblich et al. (2020) and Miles and Sefton (2020). Bottom line, our findings show that both structural change and the fall in commute costs contribute

[36] This is the limit value. In this knife-edge case of the commuting choice model used as micro-foundation, workers do not switch to faster modes at a given location with rising wages: the higher operating cost of faster commutes offsets the benefits due to a rising opportunity cost of time.

[37] In this counterfactual, the average commuting speed still increases slightly (Figure 17a): with structural change, rural workers relocated in further away suburban locations are commuting faster. Setting the elasticity of commuting costs to distance, ξ_ℓ, also to unity gets rid of this interaction between structural change and faster commutes. Without any increase in commuting speed ($\xi_w = \xi_\ell = 1$), results are however quite similar since most of the commuting cost channel is driven by the more direct income-effects of rising urban wages.

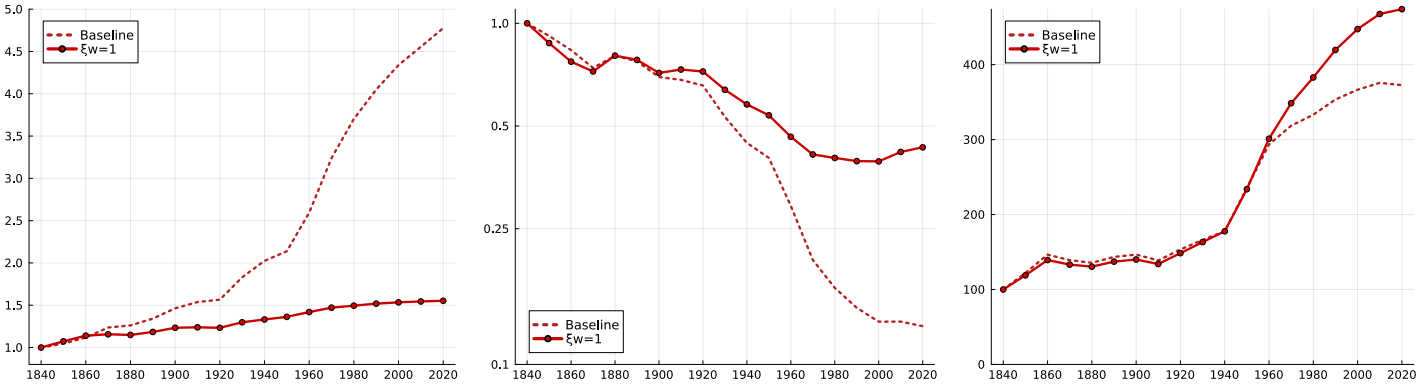
[38] For the recent period, this counterfactual generates an 'agricultural productivity gap' about twice as large as in the baseline. Fringe residents face higher commuting costs and central residents higher housing prices.



(a) Average urban density (1840=1). (b) Density at the fringe (1840=1). (c) Rental price of farmland.

Figure 16: Sensitivity to rural productivity growth.

Notes: Productivity growth in the rural sector is set to 4% of the baseline rural productivity growth (solid line), resp. 20% of the baseline (solid line with circles). Region-specific urban productivity parameters are re-estimated to preserve the distribution of city populations. Other parameters are kept to their baseline value of Table 1. Simulation for the baseline rural productivity growth is shown in dotted for comparison.



(a) Average commuting speed (1840=1). (b) Average urban density (1840=1). (c) Real Housing Price Index (1840=1).

Figure 17: Sensitivity to the elasticity of commuting costs to income.

Notes: The elasticity of commuting cost to income, ξ_w, is set to 1. All other parameters are kept to their baseline value of Table 1. Simulation for the baseline calibration shown in dotted for comparison.

crucially to the fall in average urban density. Structural change is a critical ingredient for its fall but, at the same time, without the use of faster commutes, the decline in urban density would be very short of the data.

Disentangling the effect of farmland prices on urban density. The structural change channel involves different effects: on the one hand, the price of farmland at the urban fringe (relative to income) drops and, on the other hand, the spending share on housing increases (as subsistence needs become less relevant and the rural good expenditure share falls). By limiting structural change with lower rural productivity growth, we get rid of both mechanisms. In order to pin down the aggregate effect of farmland prices on urban density, our approach is to first estimate the response of urban density to an exogenous aggregate increase in land rental prices at the fringe. Specifically, we perform a comparative static exercise where we exogenously increase the rental price of farmland by a fixed percentage in all regions at different dates $t \in \{1920, 1970, 2020\}$ relative to the baseline simulation.[39] For 2020, we find that a 10% exogenous increase in the rental price at the urban fringe of all regions increases urban density by about 3% on average—an elasticity close to the cross-sectional one (Table 2). The same elasticity is also close to 1/3 for earlier dates.

While evidence of the importance of farmland rental prices for urban density, we provide a more quantitative interpretation asking by how much urban density would have declined if aggregate farmland rental prices over household income had not dropped. We perform this exogenous counterfactual evolution in farmland prices, common across regions, over the periods 1870-1920, 1920-1970 and 1970-2020. Due to faster productivity growth and structural change, the counterfactual increase of the farmland price is an order of magnitude larger for the interim period relative to the other ones—an increase above 300% in 1970 to keep farmland prices over income constant over 1920-1970, close to 5 times (resp. 15 times) larger than the increase in 2020 (resp. 1920). Then, comparing the counterfactual change in average urban density (in log) to the baseline is suggestive of the quantitative magnitude of the farmland price mechanism. While this mechanism makes up for about 75% of the decline over 1870-1920, our counterfactual experiments suggest that it plays less of a role in the later periods—still representing a significant share, about 30% (34% over 1920-1970 and 26% over 1970-2020). The number is significantly higher in the 1870-1920 period for two reasons. First, other channels lowering density, among which faster commutes, play less of a role at the beginning of the sample than in the later periods. Subsistence needs bind more initially and individuals move less to the suburbs to enjoy large homes when urban income and commuting speed increase—mitigating the contribution of the commuting costs channel. Second, composition effects due to the reallocation of workers across cities do not vanish in the aggregate for the 1870-1920 period. As individuals move to larger and denser cities growing faster, this tends to increase average urban density over this period—both in the baseline and in the counterfactual. As a result, the remaining unexplained fall in density is particularly low over the 1870-1920 period.[40]

[39] Note that this experiment is a partial equilibrium exercise, land markets do not clear in each region k when we set exogenously the farmland price. Other model equations are left unchanged.

[40] Isolating composition effects over 1870-1920 shows that the farmland price mechanism plays almost the same role

4.6 Sensitivity and Extensions

We next investigate the robustness of the findings to some preference and technology parameters, to the presence of agglomeration/congestion forces and to a more general commuting cost specification. For sake of space, details about the computations and results for these robustness checks are relegated to Appendix B.3.

Sensitivity to preference and technology parameters. Data variations to estimate accurately the elasticity of substitution σ between urban and rural goods are lacking and we perform sensitivity with a lower (resp. a higher) values, keeping all other parameters to the baseline. Results are robust to alternative substitution patterns between both goods—the decline in average urban density being only slightly larger with a lower σ of 0.5. Using a more general CES production function in the rural sector, we also perform sensitivity with respect to the elasticity of substitution between land and labor in the rural sector, ω. Values used in the literature typically range between 0 and 1 (Bustos et al. (2016) and Leukhina and Turnovsky (2016)). The baseline assumes $\omega = 1$ and we perform sensitivity analysis with alternative values. With a lower ω, the farmland rental price (relative to income) falls more over time as land and labor are more complement in the rural sector. With a lower opportunity cost of expanding the city, the urban area increases more and the average urban density falls more—getting closer to the data.

With respect to the housing supply elasticity, we perform a sensitivity analysis assuming a constant value in the mid-range of empirical estimates, $\epsilon(\ell_k) = \epsilon_r = 3$ in all locations. Results show that keeping all parameters constant but changing the housing supply elasticity barely affects the aggregate implications. However, compared to our baseline simulation, a more elastic housing supply at the center leads to a larger provision of housing in these locations. The center is then significantly denser than in the data—the within-city density gradient becomes significantly steeper than in the data.

Congestion and Agglomeration. We extend the model to account for possible urban congestion/agglomeration forces. We consider additional urban congestion costs by assuming that commuting costs are increasing with urban population, $a(L_{u,k}) = a \cdot L_{u,k}^{\mu}$. This summarizes the potential channels through which larger cities might involve longer and slower commutes. We set externally $\mu = 0.05$ and we re-estimate the commuting cost function parameter a as well as the region-specific sectoral productivities to make sure that we shift neither the level of the commuting costs nor aggregate sectoral productivity, while still matching cross-sectional outcomes. Congestion forces reduce the expansion in area and the extent of suburbanization. By rising commuting costs, they also increase urban housing prices relative to the baseline.

We also introduce urban agglomeration forces by assuming that urban productivity increases externally with urban employment in city k, $\theta_{u,k}(L_{u,k}) = \theta_{u,k} \cdot L_{u,k}^{\lambda}$. We set $\lambda = 0.05$, in the range

as other channels lowering density—as opposed to later periods, when the commuting costs channel matters more. Composition effects being rather small post-1920, the farmland price mechanism accounts for about 30% of the fall, while other channels make up for the rest.

of empirical estimates for France (Combes et al. (2010)). We show that if one re-estimates the region-specific productivity parameters to match the data in presence of agglomeration, outcomes are virtually identical. Given that the estimation targets the urban population distribution and aggregate productivity, our results remain robust to any reasonable magnitude of agglomeration forces. Instead of targeting aggregate productivity in the estimation, we also investigate the equilibrium effects of agglomeration forces on aggregate outcomes when, for $\lambda > 0$, aggregate urban productivity increases relative to the baseline as workers move towards cities. While the equilibrium effects of agglomeration forces are important for the allocation of urban employment across cities, these effects remain small in the aggregate for the allocation across sectors—despite the very large urban expansion driven by structural change. Agglomeration forces make all cities more productive over time as workers reallocate in the urban sector. However, higher urban incomes make also rural goods more valuable increasing rural workers' wage almost one for one. General equilibrium forces thus prevent stronger worker reallocation towards the urban sector despite agglomeration benefits.

Commuting distance and residential location. Guided by the structure of French cities, our baseline results hinge on the assumption of a monocentric model where urban individuals commute to the city center to work. While endogenizing firms' location across space is beyond the scope of the paper, one can still partly relax the monocentric assumption by assuming that commuting distance at location ℓ_k in city k, $d_k(\ell_k)$, does not map one for one with residential distance ℓ_k from the central location. Using data available for the recent period to investigate the link between commuting distance and residential location (see Appendix A.5.2 for details), we find that households residing further away do commute longer distances on average. However, commuting distance increases less than one for one with the distance of residence from the city center. Moreover, individuals residing close to the center commute longer distances than the distance of their home from the central location. Lastly, data show that commuting distance increases less with the distance of residence from the center in larger cities. Based on these observations, we model commuting distance, in location ℓ_k of city k, $d_{k,t}(\ell_k)$ in a reduced-form way as follows,

$$d_{k,t}(\ell_k) = d_0(\phi_{k,t}) + d_1(\phi_{k,t}) \cdot \ell_k, \tag{26}$$

with $d_0(\phi)$ being a positive and increasing function of ϕ satisfying $\lim_{\phi \to 0} d_0(\phi) = 0$, and $d_1(\phi)$ being a decreasing function belonging to $(0, 1)$ with $\lim_{\phi \to 0} d_1(\phi) = 1$. d_0 represents the (minimum) commuting distance traveled by an individual living in the center, while d_1 is the slope between commuting distance and residential distance from the center. We set the functional forms of d_0 and d_1 under a specification that fits recent commuting data and re-estimate the commuting cost parameter a to maintain the level of commuting costs. As before, to give the best chances to this extension to match cross-sectional data while preserving aggregate structural change forces for comparison to the baseline, we re-estimate sectoral region-specific productivities holding aggregate productivity fixed. Quantitatively, cities expand more in area in the last decades in this extension, bringing the model closer to the data. As a consequence of a larger sprawling, the average urban

density falls more. This is driven by a larger fall of central density: with urban expansion, residents close to the center end up commuting larger distances—implicitly due to the reallocation of jobs away from the center—, making central locations less attractive relative to the suburbs. As a result, this extension provides a slightly better fit of cross-sectional data. Relative to the baseline, commuting distances in the center (resp. at the fringe) are larger (resp. lower) in larger cities. This, in turn, increases the area of more populated cities, reducing their average density and bringing the model closer to the data. Larger cities are still noticeably denser than in the data, but less so compared to the baseline monocentric model.

5 Conclusion

This paper develops a spatial general equilibrium model of structural change with endogenous land use and studies its implications for urbanization. We document a persistent fall of urban density in French cities since 1870 and show that the theoretical and quantitative predictions of the model are broadly consistent with the data. The quantitative version of the theory calibrated to French data explains about 70% of the urban area expansion and most of the decline in average urban density, about half of the rise in housing prices, and most of the land value reallocation from rural to urban since the mid-nineteenth century. Novel predictions regarding urban density across space line up relatively well with available data.

Agricultural productivity growth is shown to be crucial for the results, since it reduces the price of land at the urban fringe and frees up resources to be spent on housing. As a consequence, while workers reallocate away from agriculture, cities grow faster in area than in population and land prices do not rise very rapidly. Faster commuting modes also play an important and complementary role but only when combined with rural growth and structural change. When rural productivity is high, they allow households to live further away from their workplace and enjoy larger homes, contributing significantly to the decline in urban density, particularly at the city center.

Our baseline theory relies on a monocentric urban structure where all workers commute from their residential location to the center. While French cities exhibit the qualitative features of monocentric cities, such an urban structure certainly remains an approximation. Data show that commuting distance increases with residential distance to the center but less than one for one. This suggests that workers sort into jobs and residences that are closer to each other. Relaxing further the monocentric structure remains an important step to better account for the expansion of cities and the evolution of their density. We leave for future research a theory that jointly determines firms and workers location decisions across the urban space. More broadly, further heterogeneity across cities in their urban form seems necessary to account for the spatial dispersion of urban density.

Relatedly, we focus on the reallocation of economic activity from the rural to the urban sector, abstracting from the reallocation within the urban sector. We could extend our framework to consider the transition from manufactures to services in the later period. While aggregate results might not

be much affected, we believe it would matter for the cross-section of cities in recent times. Some services are provided locally, especially in large cities, implying that not all workers have to commute to the center. We also leave this extension for future research.

We also believe that our approach can be used to study the aggregate implications of policies regulating land use and urban planning. Such policies are likely to play a role in explaining the evolution of housing prices in recent years, which our current setup cannot fully replicate. To the extent that land-use policies reduce city growth on the extensive margin, they lead to greater demand for available housing units and to faster rise in their prices. The general equilibrium structure of our quantitative spatial model is well suited to conduct such policy counterfactuals. More broadly, our framework can be used to revisit a variety of normative questions in presence of externalities. While our approach is positive, urban density is at the heart of agglomeration and congestion externalities on productivity. Depending on the context, population density or urban sprawl are also sources of pollution and environmental externalities. By bringing novel insights on the determinants of urban sprawl and urban density across time and space, our approach might shed novel light on the design of policies to address such externalities.

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