# Fiscal Policy for Climate Change

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*Fiscal policy offers a number of levers to reduce carbon emissions. Climate change mitigation can for example be implemented through carbon taxation on the production or consumption side, or through debt-financed public investments in emission-reducing infrastructure. Yet these various instruments may differ significantly in their cost-effectiveness in reducing emissions and in their distributional impacts among households. We develop a macroeconomic heterogeneous-agent model with environmental externalities to address both of these questions. In this model, households derive utility from the consumption of carbon-intensive and clean goods, and from the environmental damages resulting from $CO_2$ emissions. In addition, $CO_2$ emissions affect productivity and thus relative prices. We use household data on the distribution carbon-intensive goods consumption to estimate preference parameters. Starting from a realistic fiscal structure, we then implement various tax reforms to analyze their effects on both $CO_2$ emissions and welfare along the income distribution.*

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## I. Introduction

Anthropogenic climate change has become one of the leading issues facing policymakers globally. Implementing a uniform Pigouvian tax on greenhouse gas emissions – mainly carbon dioxyde – is considered by most economists to represent the first best to address climate change (Nordhaus, 2019). Yet, enacting carbon pricing at a level commensurate with countries' emissions reduction commitments has so far proven politically difficult (Hassler, Krusell and Olovsson, 2021). In particular, in the absence of redistribution carbon taxation is usually regressive in advanced economies (Stavins, 2020). This impacts the perceived fairness and acceptability of a consumption-side carbon tax negatively (Douenne and Fabre, forth.).

This state of affairs heightens the need to examine the macroeconomic and distributional impacts of alternate climate change mitigation policies. Fiscal policy in particular offers a number of levers to tackle greenhouse gas emissions besides use-side carbon pricing. Policymakers can for example implement carbon taxation on the production side, or use public debt to finance infrastructure investments to decarbonize the most fossil-fuel intensive sectors, such as transportation and electricity production (IPCC, 2018).

In the present paper, we develop a macroeconomic heterogeneous-agent model with environmental externalities to assess the macroeconomic and distributional impacts of these alternative fiscal policy tools in mitigating climate change. We consider an economy consisting of a carbon-intensive and a clean sector, which produce a carbon-intensive and carbon-free goods respectively. A continuum of households derive utility from the consumption of these two goods and from climate-related damages resulting from greenhouse gas emissions. Importantly, the model includes a rich and realistic set of fiscal tools that can be used to set a price on the carbon externality, including consumption-side, production-side and capital taxes, and lump-sum transfers.

Our contribution relates to various recent strands of the climate macroeconomics literature. Our model builds on seminal evaluations of the optimal price of carbon such as Nordhaus (2014, 2018) and Golosov et al. (2014). In particular, we include a climate damage function directly inspired by Golosov et al. (2014). Yet this paper is even more closely related to the burgeoning literature that departs from the representative agent assumption in climate macroeconomics. In particular, we adapt the heterogeneous-agent framework in a climate change setting, in the vein of Fried (2021) and Känzig (2022). By contrast with these recent contributions, we seek to represent the full joint distribution of households' income and the carbon intensity of their final consumption. Our paper is also related to Barrage, who explores the impact of carbon taxation when governments also levy distortionary taxes, particularly on capital income. However this assessment is conducted in a representative agent framework and abstracts from distributional considerations.

We build a full-fledged heterogeneous-agent model, which reproduces both income and wealth inequality and the heterogeneity in the consumption of carbon-intensive (brown) goods in total consumption. To account for the observed heterogeneity in this new dimension, we then estimate the parameters of the utility function using the Simulated Method of Moment to match the empirical distribution (by decile) of brown goods. This is necessary to yield a quantitatively relevant welfare effect of any policy to reduce carbon emission.

To calibrate this distribution, we build a detailed estimate of households direct emissions in the United States by income decile. We combine household-level fossil-intensive energy expenditure obtained from the Consumer Expenditure Survey with state-level prices and carbon intensity factors for each energy vector.

In the current version of this paper, we perform policy experiments to assess the change in equilibrium inequality and carbon emissions for various simple policies. In future revisions, we will apply advanced modelling tools to derive optimal policies in heterogeneous agent models, building on LeGrand and Ragot (2021). This 'truncation method' to solve heterogeneous agents models allows us to derive the optimal time-varying path and allocation of taxes and public debt to reduce carbon emission, while minimizing the welfare cost of the transition and accounting for inequality dynamics.

The paper is laid out as follows. Section II presents the model and details its specification. Section III describes the calibration of the households' consumption share of the carbon intensive good, model parameters, the baseline fit of our model to the calibration data. Section IV presents comparative statics results when modeling counterfactual scenarios on the price of the clean and carbon-intensive goods.

## II. A heterogeneous-agent model with environmental externalities

We consider a discrete time-economy populated a continuum of agents with unit mass. Agents are distributed on an interval $\mathcal{I}$ according to measure $\ell$. The law of large numbers is assumed to hold (see Green, 1994).

### *A. Production*

The economy features one final consumption good that consumes capital, labor and energy. Energy is produced in a energy sector, which solely relies on capital and labor. Finally, households consume both the final good and energy separately. Each sector generates carbon emissions and carbon taxation is then applied as follows to each actor:

- *Energy sector*: to emissions it generates in the production of energy (both fossil and electricity);
- *Final good sector*: to emissions resulting from burning fossil energy and industrial processes;
- *Households*: to emissions resulting from burning fossil energy directly (natural gas, gasoline and diesel mainly).

We will use the following subscripts: $h$ for households, $e$ for energy ad $f$ for the final good sector.

The pre-tax price of energy is denoted by $\tilde{p}_{e,t}$, while the pre-tax price of the final good is $\tilde{p}_{f,t}$. The pre-tax labor wage is $\tilde{w}_t$. There is no tax on capital and the capital rate is denoted by $r_t$.

### Energy sector

Energy is produced by a representative firm endowed with a Cobb-Douglas production function featuring constant returns to scale. We assume the presence of climate damages $D_t$ that affect the sector's productivity. Denoting by $\alpha_e$ the capital share and $A_{e,t}$ the sector's productivity in the absence of damage, the energy production can be written as:

$$Y_{e,t} = \bar{A}_{e,t} K_{e,t-1}^{\alpha_e} L_{e,t}^{1-\alpha_e}, \tag{1}$$

$$\bar{A}_{e,t} = A_{e,t}(1 - D_{e,t}). \tag{2}$$

A negative externality of energy production is the production of emissions $m_{e,t}$ from burning (fossil) energy and from its own industrial processes with intensity $\phi_e$. The energy sector can then invest in abatement $\mu_{e,t}$ to reduce its own emissions. Formally, the emissions of the energy sector are:

$$m_{e,t} = (1 - \mu_{e,t})\phi_e Y_{e,t}. \tag{3}$$

We assume a general abatement-as-expenditure formulation, with an abatement cost curve specific to the energy good. We assume that the cost curve is labeled in terms of final goods and is a function of $\mu_{e,t}$ and total output $Y_{e,t}$ and is denoted by $g_e(\mu_{e,t}, Y_{e,t})$, which nests the specification of (see ?).

The energy sector pays a sector-specific carbon tax $\tau_{e,t}$ that is proportional to the firm's emission $m_{e,t}$. Overall, the firm's objective writes as follows:

$$\max_{\{K_{e,t},L_{e,t},\mu_{e,t}\}} \tilde{p}_{e,t}Y_{e,t} - (r_t + \delta_e)K_{e,t-1} - \tilde{w}_t L_{e,t} - \tau_{e,t}m_{e,t} - \tilde{p}_{f,t}g_e(\mu_{e,t}, Y_{e,t}), \tag{4}$$

or

$$\max_{\{K_{e,t},L_{e,t},\mu_{e,t}\}} (\tilde{p}_{e,t} - \tau_{e,t}(1-\mu_{e,t})\phi_e)Y_{e,t} - (r_t + \delta_e)K_{e,t-1} - \tilde{w}_t L_{e,t} - \tilde{p}_{f,t}g_e(\mu_{e,t}, Y_{e,t}). \tag{5}$$

The firm's profit maximization then implies that the abatement solves the following equation:

$$\tilde{p}_{f,t}\frac{\partial g_e}{\partial \mu_{e,t}}(\mu_{e,t}, Y_{e,t}) = \tau_{e,t}\phi_e Y_{e,t}. \tag{6}$$

and hence after substitution the following factor prices:

$$r_t = \alpha_e p_{e,t}(1-D_t)A_{e,t}K_{e,t-1}^{\alpha_e-1}L_{e,t}^{1-\alpha_e} - \delta_e, \tag{7}$$

$$\tilde{w}_t = (1-\alpha_e)p_{e,t}(1-D_t)A_{e,t}K_{e,t-1}^{\alpha_e}L_{e,t}^{-\alpha_e}, \tag{8}$$

$$\text{where: } p_{e,t} = \tilde{p}_{e,t} - \tau_{e,t}(1-\mu_{e,t})\phi_e - \tilde{p}_{f,t}\frac{\partial g_e}{\partial Y_{e,t}}(\mu_{e,t}, Y_{e,t}). \tag{9}$$

Depending on the functional form of the abatement, there is possibly a profit equal to:

$$\Pi_{e,t} = p_{e,t}Y_{e,t} - (r_t + \delta_e)K_{e,t-1} - \tilde{w}_t L_{e,t} - \tilde{p}_{f,t}g_e(\mu_{e,t}, Y_{e,t}) + \tilde{p}_{f,t}\frac{\partial g_e}{\partial Y_{e,t}}(\mu_{e,t}, Y_{e,t})Y_{e,t}.$$

Using constant returns-to-scale, we have

$$p_{e,t}Y_{e,t} = (r_t + \delta)K_{e,t-1} + \tilde{w}_t L_{e,t},$$

which implies the following profit expression:

$$\Pi_{e,t} = \tilde{p}_{f,t}\frac{\partial g_e}{\partial Y_{e,t}}(\mu_{e,t}, Y_{e,t})Y_{e,t} - \tilde{p}_{f,t}g_e(\mu_{e,t}, Y_{e,t}),$$

which is assumed to be fully taxed away.

### Final good sector

The final good sector is very similar to the energy sector, except that it uses energy for production in addition to capital and labor. The capital share is denoted $\alpha_f$, the labor share $\alpha_l$, the no-damage productivity $A_{f,t}$ and the depreciation $\delta_l$. The climate change damages also apply to the final good sector. The final good production can be written as:

$$Y_{f,t} = (1 - D_t)A_{f,t}K_{f,t-1}^{\alpha_f}L_{f,t}^{\alpha_l}E_{f,t}^{1-\alpha_f-\alpha_l}. \tag{10}$$

Damages are the same in both sectors. The final good sector can also abate, but its portfolio of abatement technology options is different from that of the energy sector (e.g. efficient industrial boilers are part of the final good sector's abatement curve, but not of the energy sector's). With energy emission intensity $\phi_f$ and abatement $\mu_{f,t}$, its emissions are therefore:

$$m_{f,t} = (1 - \mu_{f,t})\phi_f Y_{f,t}. \tag{11}$$

The abatement cost curve is specific to the final good sector and denoted by $g_f(\mu_{f,t}, Y_{f,t})$ and the firm also pays a sector-specific carbon tax $\tau_{f,t}$. The firm's objective write as follows:

(12)

$$\max_{\{K_{f,t},L_{f,t},\mu_{f,t}\}} \tilde{p}_{f,t}Y_{f,t} - (r_t + \delta_f)K_{f,t-1} - w_tL_{f,t} - \tilde{p}_{e,t}E_{f,t} - \tau_{f,t}m_{f,t} - \tilde{p}_{f,t}g_f(\mu_{f,t}, Y_{f,t})$$

The firm's profit maximization then implies that the abatement solves the following equation:

$$\tilde{p}_{f,t}\frac{\partial g_f}{\partial \mu_{f,t}}(\mu_{f,t}, Y_{f,t}) = \tau_{f,t}\phi_f Y_{f,t}. \tag{13}$$

and hence after substitution the following factor prices:

$$r_t = p_{f,t}\alpha_f(1-D_t)A_{f,t}K_{f,t-1}^{\alpha_f-1}L_{f,t}^{\alpha_l}E_{f,t}^{1-\alpha_f-\alpha_l} - \delta_f, \tag{14}$$

$$\tilde{w}_t = p_{f,t}\alpha_l(1-D_t)A_{f,t}K_{f,t-1}^{\alpha_f}L_{f,t}^{\alpha_l-1}E_{f,t}^{1-\alpha_f-\alpha_l}, \tag{15}$$

$$\tilde{p}_{e,t} = p_{f,t}(1-\alpha_f-\alpha_l)(1-D_t)A_{f,t}K_{f,t-1}^{\alpha_f}L_{f,t}^{\alpha_l}E_{f,t}^{-\alpha_f-\alpha_l}, \tag{16}$$

$$\text{where: } p_{f,t} = \tilde{p}_{f,t} - \tau_{f,t}(1-\mu_{f,t})\phi_f - \tilde{p}_{f,t}\frac{\partial g_f}{\partial Y_{f,t}}(\mu_{f,t}, Y_{f,t}). \tag{17}$$

Again, the final sector may generate a profit equal to:

$$\Pi_{f,t} = \tilde{p}_{f,t}\frac{\partial g_f}{\partial Y_{f,t}}(\mu_{f,t}, Y_{f,t})Y_{f,t} - \tilde{p}_{f,t}g_f(\mu_{f,t}, Y_{f,t}),$$

that is, as the energy-sector profits fully taxed away.

### *B. Households*

The economy is populated by a unit mass of households, who face an uninsurable income risk. The income process $y$ takes $n$ possible distinct values, denoted $y_1, \ldots, y_N$ and follows a first-order Markov chain with a constant transition matrix $\Pi$. We denote by $s_k$ the share of agents endowed with income $y_k$ – where $\sum_{k=1}^N s_k = 1$. The vector $\boldsymbol{s} = (s_1, \ldots, s_N)$ corresponds to the stationary probability associated to matrix $\Pi$: $\boldsymbol{s}\Pi = \boldsymbol{s}$.

We normalize the labor supply of each household to one, such that the aggregate labor supply $\overline{L}$ verifies:

$$\overline{L} = \sum_{k=1}^N s_k y_k.$$

### Household types

Besides ex-post heterogeneity related to income risk realization, we assume that households differ according to a type $\theta$ that affects their preferences. The type of an agent is exogenous, fixed over time and picked from a finite set $\Theta$. This preference heterogeneity may reflect heterogeneity in choice of residential location and taste for brown goods.

### Preferences

Households have time-additive preferences and per period utility is discounted with a common discount factor $\beta \in (0,1)$. In each period, households derive utility from consumption of energy and of the consumption good, denoted by $c_e$ and $c_f$ respectively. Both energy and the final consumption goods are partial substitutes and aggregated through a CES aggregator, which is household type-specific. For a household of type $\theta$, the share parameters are denoted by $\omega_{e,\theta}, \omega_{f,\theta} \in [0,1]$ (with $\omega_{e,\theta} + \omega_{f,\theta} = 1$), while the elasticity of substitution is equal to $(1-\alpha_\theta)^{-1}$, with $\alpha_\theta < 1$. The aggregation also features subsistence consumption levels $\bar{c}_{f,\theta} \geq 0$ and $\bar{c}_{e,\theta} \geq 0$. The aggregate consumption good $C_\theta(c_f, c_e)$ can then be formally expressed as:

(18)

$$C_\theta(c_f, c_e) = \begin{cases} \left(\omega_{f,\theta}^{1-\alpha_\theta} \left(c_f - \bar{c}_{f,\theta}\right)^{\alpha_\theta} + \omega_{e,\theta}^{1-\alpha_\theta} \left(c_e - \bar{c}_{e,\theta}\right)^{\alpha_\theta}\right)^{\frac{1}{\alpha_\theta}} & \text{if } \alpha_\theta < 1 \text{ and } \alpha_\theta \neq 0, \\ \omega_{f,\theta} \ln\left(c_f - \bar{c}_{f,\theta}\right) + \omega_{e,\theta} \ln\left(c_e - \bar{c}_{e,\theta}\right) & \text{if } \alpha_\theta = 0. \end{cases}.$$

Instantaneous utility $U_\theta(c_f, c_e)$ is assumed to be defined over this aggregate good:

$$U_\theta(c_f, c_e) = u\left(C_\theta(c_f, c_e)\right),$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave and independent of agent's types. For the sake of simplicity, we assume that intertemporal elasticity of aggregate consumption is constant and equal to $\sigma^{-1} > 0$, such that $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$.

### Emissions.

The households' consumption also generate emissions that are assumed to be directly proportional to their consumption of energy. Their abatement curve is yet again specific to households and denoted by $\mu_{h,t}$. The emissions $m_{h,t}$ of an household consuming the energy amount $c_{e,t}$ have specific intensity $\phi_h$ and can be written as:

$$m_{h,t} = (1 - \mu_{h,t})\phi_h c_{e,t}. \tag{19}$$

The specificity is that households' abatement is common to all households and determined by the government's investing in abating them.

### Households' program

We consider a household of type $\theta$, currently endowed with beginning-of-period wealth $a$ and labor income $y$. They also receive the lump-sum tax $T$ and pay the carbon tax $\tau_h$ proportional to their emissions. They have to decide how much energy and consumption goods to consume and how much to save, subject to their budget constraint and a credit limit preventing them from borrowing.

We take advantage of the results of to express the households' program in recursive form. We denote by $V_\theta(a, y)$ the value function of the household of type $\theta$, beginning-of-period wealth $a$, and income $y$. Formally, we have:

$$V_\theta(a, y) = \max_{(c_f, c_e, a')} u\left(C_\theta(c_f, c_e)\right) + \beta \mathbb{E}_{y'}\left[V_\theta\left(a', y'\right)\right], \tag{20}$$

$$\text{subject to } a' = Ra + wy + T - \tilde{p}_f c_f - (\tilde{p}_e + (1 - \mu_h)\phi_h \tau_h) c_e, \tag{21}$$

$$a' \geq 0, \tag{22}$$

$$c_f, c_e > 0, \tag{23}$$

where $\mathbb{E}_{y'}$ is the expectation over future income realizations $y'$. Note that households do not choose their abatement $\mu_h$ that is decided at the governmental level and valid for all households.

We denote by $\lambda$ and $\mu$ the Lagrange multipliers on the budget constraint (21) and the credit constraint (22), respectively. Combining the first-order condition and the envelop conditions on $a$ yields the following Euler equation on the Lagrange multiplier $\lambda$:

$$\lambda = \beta \mathbb{E}\left[R'\lambda'\right] + \kappa. \tag{24}$$

The first-order conditions on green and brown consumption choices imply:

$$\lambda = \frac{1}{\tilde{p}_f}\frac{\partial C_\theta(c_f, c_e)}{\partial c_f} u'\left(C_\theta(c_f, c_e)\right) = \frac{1}{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}\frac{\partial C_\theta(c_f, c_e)}{\partial c_e} u'\left(C_\theta(c_f, c_e)\right), \tag{25}$$

which, after some algebra, is equivalent to:

$$\lambda = \frac{1}{\tilde{p}_f}\omega_{f,\theta}^{1-\alpha_\theta}\left(c_f - \bar{c}_{f,\theta}\right)^{\alpha_\theta - 1} C_\theta(c_f, c_e)^{1-\alpha_\theta-\sigma}, \tag{26}$$

$$c_e - \bar{c}_{e,\theta} = \frac{\omega_{e,\theta}}{\omega_{f,\theta}}\left(\frac{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}{\tilde{p}_f}\right)^{\frac{1}{\alpha_\theta - 1}}(c_f - \bar{c}_{f,\theta}), \tag{27}$$

Given the number of possible combinations implied by the Euler equation (24) and equalities (26) and (27), it is simpler to use one intertemporal equation and two static ones. Our choice enables us to follow the dynamics of the consumption good, from which we deduce the consumption of energy.

### Simplifying the household's program

We can further simplify the model dynamics by expressing it solely as a function of final consumption good $c_f$. The idea is to use equation (27) to substitute for the expression of $c_e$. First, the budget constraint (21) becomes:

$$a' = Ra + wy + T - (\tilde{p}_e + \tau_h(1-\mu_h)\phi_h)\left(\bar{c}_{e,\theta} - \frac{\omega_{e,\theta}}{\omega_{f,\theta}}\left(\frac{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}{\tilde{p}_f}\right)^{\frac{1}{\alpha_\theta - 1}}\bar{c}_{f,\theta}\right)$$
$$- c_f\left(\tilde{p}_f + (\tilde{p}_e + \tau_h(1-\mu_h)\phi_h)\frac{\omega_{e,\theta}}{\omega_{f,\theta}}\left(\frac{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}{\tilde{p}_f}\right)^{\frac{1}{\alpha_\theta - 1}}\right)$$

or equivalently:

(28)

$$a' = Ra + wy + \hat{T} - \hat{p}_f c_f,$$

(29)

$$\text{where: } \hat{T} = T - (\tilde{p}_e + \tau_h(1-\mu_h)\phi_h)\left(\bar{c}_{e,\theta} - \frac{\omega_{e,\theta}}{\omega_{f,\theta}}\left(\frac{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}{\tilde{p}_f}\right)^{\frac{1}{\alpha_\theta - 1}}\bar{c}_{f,\theta}\right),$$

(30)

$$\text{and } \hat{p}_f = \tilde{p}_f + (\tilde{p}_e + \tau_h(1-\mu_h)\phi_h)\frac{\omega_{e,\theta}}{\omega_{f,\theta}}\left(\frac{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}{\tilde{p}_f}\right)^{\frac{1}{\alpha_\theta - 1}}.$$

Second, equation (26) characterizing $\lambda$ becomes:

$$\lambda = \frac{1}{\tilde{p}_f}\omega_{f,\theta}^{1-\alpha_\theta}\left(\omega_{f,\theta}^{1-\alpha_\theta} + \omega_{e,\theta}^{1-\alpha_\theta}\left(\frac{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}{\tilde{p}_f}\frac{\omega_{f,\theta}^{1-\alpha_\theta}}{\omega_{e,\theta}^{1-\alpha_\theta}}\right)^{\frac{\alpha_\theta}{\alpha_\theta - 1}}\right)^{\frac{1-\alpha_\theta-\sigma}{\alpha_\theta}}(c_f - \bar{c}_{f,\theta})^{-\sigma},$$
$$= \frac{1}{\tilde{p}_f}\omega_{f,\theta}^{\sigma}\left(\omega_{f,\theta} + \omega_{e,\theta}\left(\frac{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}{\tilde{p}_f}\right)^{\frac{\alpha_\theta}{\alpha_\theta - 1}}\right)^{\frac{1-\alpha_\theta-\sigma}{\alpha_\theta}}(c_f - \bar{c}_{f,\theta})^{-\sigma},$$

which can be plugged into the Euler equation (24). For credit-constrained households, we have:

$$a' = 0. \tag{31}$$

At the steady-state equilibrium, prices are constant. Assuming $\bar{c}_{f,\theta} = 0$, the solution to the agent's program can be simplified as follows:

$$c_f^{-\sigma} = \beta \mathbb{E}\left[R'(c_f')^{-\sigma}\right] + \kappa, \tag{32}$$

$$a' = Ra + wy + \hat{T} - \hat{p}_f c_f \geq 0, \tag{33}$$

$$\text{where: } \hat{T} = T - (\tilde{p}_e + \tau_h(1-\mu_h)\phi_h)\, \bar{c}_{e,\theta}, \tag{34}$$

$$\text{and } \hat{p}_f = \tilde{p}_f + (\tilde{p}_e + \tau_h(1-\mu_h)\phi_h) \left( \frac{\tilde{p}_e + \tau_h(1-\mu_h)\phi_h}{\tilde{p}_f} \frac{\omega_{f,\theta}}{\omega_{e,\theta}} \right)^{\frac{1}{\alpha_\theta - 1}}. \tag{35}$$

### C. *Government and market clearing*

#### Government.

We consider a benevolent government that can influence $CO_2$ atmospheric emissions through $CO_2$ taxation, labor tax, and lump-sum transfers. The government revenues also include the profits of the two sectors. We also assume the presence of exogenous public spending. Note that the government can choose negative taxes, and hence to subsidize one sector of the economy. The $CO_2$ are sector-specific and concern households, energy sector and final good sector. The governmental budget constraint can be written as:

$$\Pi_e + \Pi_f + \tau_h \int m_h(a,y)\Lambda(da,dy) + \tau_e m_e + \tau_f m_f + \tau_L \overline{L} + \tau_K (r_t - \delta) K = G + T + g_h(\mu_h, \int c_e(a,y)\Lambda(da,dy)).$$

The stock of emissions is:

$$m = m_f + m_e + \int m_h(a,y)\Lambda(da,dy)$$

#### Market clearing.

The labor market clearing implies:

$$\overline{L} = \sum_{k=1}^{N} s_k y_k = L_e + L_f.$$

We denote by $\Lambda : [0,\infty) \times \{y_1, \ldots, y_N\}$ the distribution of agents over the state space, equal to the Cartesian product of the asset and incomes spaces.[1] The financial market clearing condition implies that aggregate savings should equal total capital. Formally:

$$K' = K'_e + K'_f = \int a'(a,y)\Lambda(da,dy),$$

where $a'(a,y)$ is the end-of-period savings policy function solving the households' program (20)–(23).

Market clearing for energy can be written as follows:

$$Y_e = E + \int c_e(a,y)\Lambda(da,dy),$$

where $E$ is the quantity of energy consumed in the final good production and $c_e(a,y)$ is the policy function for energy consumption.

For final good consumption goods, we similarly have:

$$G+K'+\int c_f(a,y)\Lambda(da,dy)+g_h(\mu_h, \int c_e(a,y)\Lambda(da,dy))+g_e(\mu_e, Y_{e,t})+g_f(\mu_f, Y_{f,t}) = (1-\delta)K+Y_f$$

At the steady state, with no abatement cost, this simplifies into:

$$G + \delta K + C_f = Y_f$$

**Proof:**

The aggregation of individual budget constraint yields

$$K' = (1+(1-\tau_K)\,r)\,K + w\overline{L} + T - \int c_f(a,y)\Lambda(da,dy) - \tilde{p}_e \int c_e(a,y)\Lambda(da,dy) - \tau_h \int m_h(a,y)\Lambda(da,dy$$

[1]The existence of $\Lambda$ is proved in

while the government budget constraint is

$$G + T + g_h(\mu_h, \int c_e(a,y)\Lambda(da,dy)) = \tau_h \int m_h(a,y)\Lambda(da,dy) + \tau_e m_e + \tau_f m_f + \tau_L w\overline{L} + \tau_K r_t K$$
$$+ \frac{\partial g_f}{\partial Y_{f,t}}(\mu_{f,t}, Y_{f,t})Y_{f,t} - g_f(\mu_{f,t}, Y_{f,t}) + \frac{\partial g_e}{\partial Y_{e,t}}(\mu_{e,t}, Y_{e,t})Y_{e,t} - g_e(\mu_{e,t}, Y_e$$

Summing the two implies

(36)

$$G + K' + \int c_f(a,y)\Lambda(da,dy) + g_h(\mu_h, \int c_e(a,y)\Lambda(da,dy)) = (1+r)K + \tilde{w}\overline{L} - \tilde{p}_e \int c_e(a,y)\Lambda(da,dy)$$
$$+ \tau_e m_e + \tau_f m_f$$
$$+ \frac{\partial g_f}{\partial Y_{f,t}}(\mu_{f,t}, Y_{f,t})Y_{f,t} - g_f(\mu_{f,t}, Y_{f,t}) + \frac{\partial g_e}{\partial Y_{e,t}}(\mu$$

From constant-returns-to-scale production functions, we have:

$$p_{f,t}Y_{f,t} = (r_t + \delta)K_{f,t-1} + \tilde{w}_t L_{f,t} + \tilde{p}_{e,t}E_{f,t}$$
$$p_{e,t}Y_{e,t} = (r_t + \delta)K_{e,t-1} + \tilde{w}_t L_{e,t}$$

Hence

$$(r + \delta)K + \tilde{w}\overline{L} = p_f Y_f + p_e Y_e - \tilde{p}_e E_f$$

Plugging this into (36) yields:

(37)

$$G + K' + \int c_f(a,y)\Lambda(da,dy) + g_h(\mu_h, \int c_e(a,y)\Lambda(da,dy)) = (1-\delta)K - \tilde{p}_e(\int c_e(a,y)\Lambda(da,dy) + E_f - Y$$
$$+ p_f Y_f + \tau_f m_f + \frac{\partial g_f}{\partial Y_{f,t}}(\mu_{f,t}, Y_{f,t})Y_{f,t}$$
$$+ p_e Y_e - \tilde{p}_e Y_e + \tau_e m_e + \frac{\partial g_e}{\partial Y_{e,t}}(\mu_{e,t}, Y_{e,t})Y_{e,t}$$
$$- g_f(\mu_{f,t}, Y_{f,t}) - g_e(\mu_{e,t}, Y_{e,t})$$

Using the expressions of $p_e$ and $p_f$ that imply $p_e Y_e = \tilde{p}_e Y_e - \tau_e m_e - \frac{\partial g_e}{\partial Y_e}(\mu_e, Y_e) Y_e$ and $p_f Y_f = Y_f - \tau_f m_f - \frac{\partial g_f}{\partial Y_f}(\mu_f, Y_f) Y_f$, we deduce:

(38)

$$G + K' + \int c_f(a,y)\Lambda(da,dy) + g_h(\mu_h, \int c_e(a,y)\Lambda(da,dy)) = (1-\delta)K - \tilde{p}_e(\int c_e(a,y)\Lambda(da,dy) + E_f - Y$$
$$+ Y_f - g_f(\mu_{f,t}, Y_{f,t}) - g_e(\mu_{e,t}, Y_{e,t})$$

The clearing on energy market implies $\int c_e(a,y)\Lambda(da,dy) + E = Y_e$ and:

$$G + \int c_f(a,y)\Lambda(da,dy) + K' + g_h(\mu_h, \int c_e(a,y)\Lambda(da,dy)) + g_f(\mu_{f,t}, Y_{f,t}) + g_e(\mu_{e,t}, Y_{e,t}) = (1-\delta)K + Y_f.$$

### *D. Simulation*

The government chooses a tuple $(\tau_h, \mu_h, \tau_e, \tau_f, \tau_K, T)$, solve for the individual program and iterate on $T$ until (**??**) holds.

## III. Model calibration

### *A. Distribution of households greenhouse gas emissions*

To calibrate the distribution of household carbon emissions as a function of income, following Levinson and O'Brien (2019) and Sager (2019), we obtain data on both household expenditure on each good $k$, $c_{i,k}$, and their associated greenhouse gas emissions intensities $e_k$ in $\mathrm{kgCO_{2,eq}}$ per dollar:

$$m_i = \sum_k c_{i,k} e_k \tag{39}$$

Households' consumption emits GHG through two channels: either through the direct combustion of fossil fuels for energy-related use (*direct* emissions) or through the emissions embedded in the production of the goods and services they purchase (*indirect* emissions).

In the following, we focus on carbon taxation targeting households' direct emissions stemming from their consumption of petroleum-derived fuels (gasoline and diesel in particular), natural gas, electricity and coal. These are the most easily targeted through carbon pricing, as the carbon intensity of these energy goods can be more readily measured. Future revisions of the present paper will extend the estimation of households' carbon footprint to indirect emissions.

We estimate the distribution of direct household emissions in the United States in 2019. We obtain data on household's energy-related expenditure from the Consumer Expenditure Survey for the three fossil fuels aforementioned and electricity. To construct emission factors in monetary terms, we combine physical emission intensities expressed in $kgCO_{2,eq}$ per energy unit (MMBtu or kWh) with energy price data.

The emission intensity of each fossil fuel, while slightly varying by grade in the case of petroleum-derived fuels, can be considered homogeneous across the US. We therefore obtain standard emission factors from the US Environmental Protection Agency. However, the carbon intensity of electricity is a direct function of the local power mix, which is highly heterogeneous across state boundaries in the US. To correctly account for this variance, we obtain data on state-level average carbon intensity per kWh of electricity supplied from the US Department of Energy.

We finally complement these physical emission factors with state-level energy prices for the five main vectors accounted for in our expenditure dataset: electricity, gasoline, diesel, natural gas and coal. We obtain price information in dollars per MMBtu from the US DoE State Energy Data System. The use of state-level data allows us to further account for the high underlying spatial variance in energy pricing: as an example, average electricity prices in 2019 in the continental US ranged from \$0.07/kWh in Louisiana to \$0.18/kWh in Rhode Island. The combination of physical emission intensities and energy price data allows us to compute the whole set of $e_k$ at the state level. Figure 1 illustrates the spatial heterogeneity in emission intensity that our approach allows to recover.

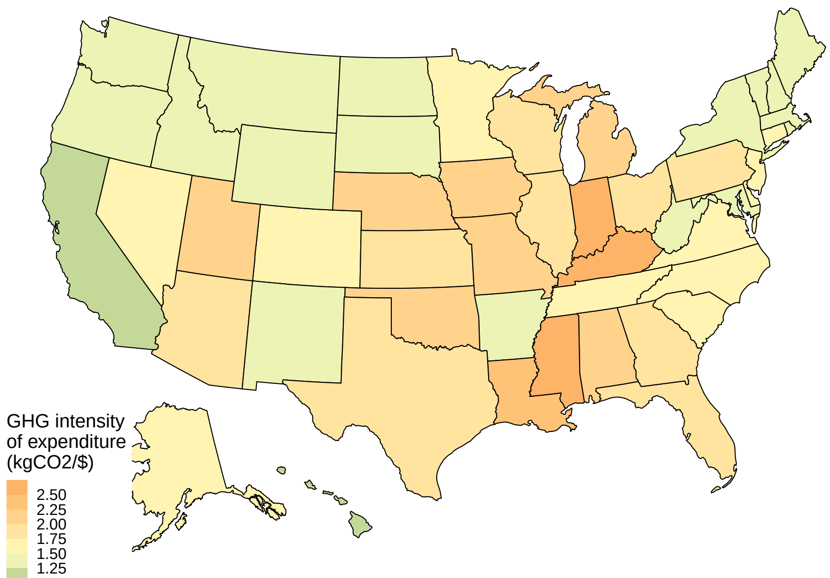


FIGURE 1. DIRECT EMISSIONS INTENSITY OF ENERGY EXPENDITURE BY STATE

This methodology allows us to compute emission intensity at the household level. Using CEX sampling weights, we construct an environmental Engels curve for direct GHG emissions by recovering emission intensities by total expenditure decile.

### *B. Macroeconomic calibration*

We use Exiobase 3 to calibrate the main parameters describing the final goods and energy sectors, along with their associated greenhouse gas emissions. Exiobase 3 is a multi-regional input-output database that provides comprehensive and consistent data on the economic activities of countries around the world. It is widely used by researchers to analyze the global economy and understand interdependencies between countries, and is increasingly used in the macro-environmental literature .

Exiobase 3 is built on a detailed and consistent set of data sources, including official statistics and expert estimates. It covers a wide range of economic sectors,

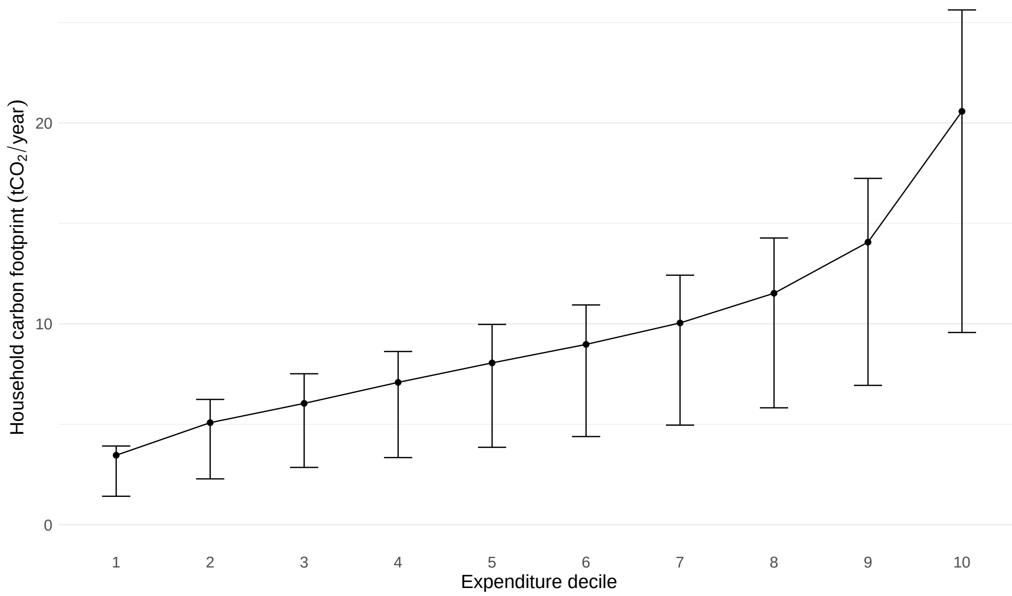


Figure 2. U.S. household direct emissions by decile of expenditure

including agriculture, manufacturing, and services, and includes both domestic and international trade. The database is also designed to be flexible, allowing users to analyze different scenarios and conduct sensitivity analyses. In addition, Exiobase 3 includes a number of tools and resources to help users understand and interpret the data, such as sectoral and regional classifications, data quality indicators, and documentation on the methodology and assumptions underlying the estimates.

### *C. Model parameters*

The main feature of this heterogeneous agent model is its capacity to generate a realistic level of income inequality and, most importantly, to replicate the consumption share of green and brown goods in the income distribution. This last property is key to match households' exposure to any change in the price of carbon in the economy. Our empirical strategy is thus to estimate the utility function of households to reproduce the share of consumption of green and brown goods in the economy.

Using the data of Figure 2, we first compute the share of brown goods in the total expenditure of households, ranked by expenditure decile. The data is reported in Figure 3, in red dots, on the left hand side. This graph shows that the consumption share of brown goods in the first expenditure decile is roughly 12% and goes down monotonically along the income distribution to fall to 6% in the last decile. This known property (see Känzig (2022) among others) implies that, although the absolute level of expenditure on brown goods increases with total expenditure, its share of total expenditure falls along the income distribution.

To replicate this distribution of brown goods consumption in the model, we implement a Simulated Method of Moments to reproduce 11 moments: the ten share of expenditure on brown goods by decile and the total share of aggregate consumption of green goods.

We first calibrate the standard parameters of the model. As a benchmark, we first normalize post-tax prices of each goods to 1 (which equivalently provides a normalization of the utility function). Second, the discount factor is set to $\beta = 0.96$ to match annual data. The real interest rate is set to $r = 4\%$, which is the standard value to match a realistic level of saving over total income. The real wage is set to 1 as a normalization of income. The overall utility curvature is to $\sigma = 2$, and the intertemporal elasticity of substitution to 0.5, which is a realistic value used in the literature. We will perform sensitivity test for these calibrated parameters in future revisions of the present paper. Following Castaeneda, Diaz-Gimenez and Rios-Rull (2003), the persistence of the uninsurable idiosyncratic income process $\rho$ and the standard deviation of the income process $\epsilon$ are set to match realistic equilibrium inequality in wealth. At this stage, we target a Gini coefficient of wealth of roughly 0.7. We find $\rho = 0.97$. and $\epsilon = 0.038$, which are values which are consistent with empirical estimate of Krueger, Mitman and Perri (2018).

We are left with three parameters to estimate : the brown minimal consumption

FIGURE 3. BASELINE MODEL FIT

$\bar{c}$, the elasticity of substitution across goods $\alpha_\theta$, the weight of brown and green goods in the utility function (which sum to 1), $\omega_{G,\theta}$ and $\omega_{B,\theta}$. Let's consider the set of estimated parameters $p = (\bar{c}, \alpha_\theta, \omega_{B,\theta})$, $m_k$ the $k = 1...11$ moments we try to match, and $\hat{m_k}(p)$ the model-generated $k$-moment when the parameters are $p$. The estimated parameters are then the solution to:

$$\min_p \sum_{k=1}^{11} \left(\hat{m}_k(p) - m_k\right)^2$$

We use the identity matrix to weigh the moments. The estimated parameters are provided in table 1, and the estimation results can be seen in Figure 3, where the stars indicate the model outcome for the estimated coefficients. The equilibrium of the model is the steady-state distribution of agents together with the equilibrium distribution of consumption. From this, we can derive the model counterpart of the moments of the data. The model does a good job in reproducing the decreasing share of consumption of brown goods over the income distribution. The fit is not perfect at the top decile. We are currently improving this fit by introducing an empirical weighting matrix in the estimation.

## IV. Counterfactual scenarios

Given this estimation, we now perform a number of policy experiments to observe the change in consumption inequality and total $CO_2$ emissions as measured by the overall consumption of brown goods. In the current iteration of the present paper, these experiments are performed in partial equilibrium to observe the change in the equilibrium distribution of agents – this restriction will be lifted in future revisions. More precisely, we compute the equilibrium distribution of the heterogeneous-agent model for each parameter change. Figure 4 plots the long-run effect of an increase the price of brown good by 15%, which can be seen as an increase of a carbon tax by 15%.

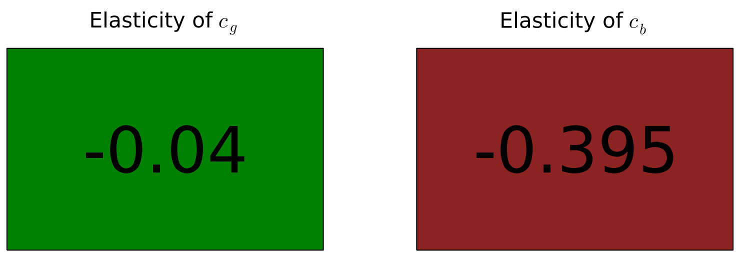


FIGURE 4. INCREASING THE BROWN GOOD PRICE BY 15%.

On can observe that the whole share of brown good consumption decreases by roughly 10 %. The right-hand side of the figure also shows that the consumption of green goods decreases because of a negative uncompensated wealth effect, which is due to an increase in prices.

Next, we focus on decreasing the price of the green good by 15 %. Interestingly, the effect is very different as the distribution of brown good expenditure barely changes. This is due the fact that all expenditure deciles increase their consumption of green goods in equal proportion. This asymmetry between the change in price of the green and brown goods stems from the existence of a minimal consumption need for brown goods, $\bar{c}$, which is necessary to reproduce the empirical decreasing share of brown goods as a function of income.

Our final experiment is a decrease in wage by 15%, presented in Figure 6. This last experiment simulates an increase in labor tax to finance some public investment to mitigate the effects of carbon emissions. This results in an overall increase in the share of brown consumption, which is due a bigger fall in the consumption

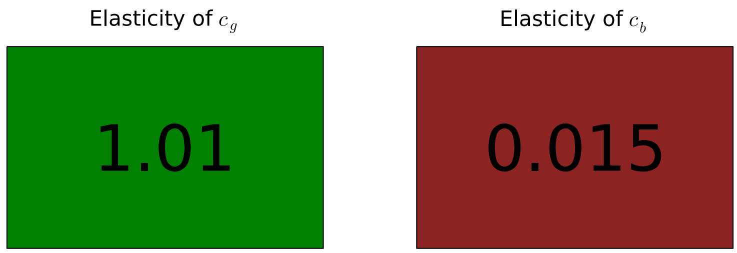


FIGURE 5. DECREASING THE GREEN GOOD PRICE BY 15%.

of green goods relative to brown goods, as can be seen at the right of the Figure. This outcome is again a direct consequence of the minimum consumption of brown goods, which makes its consumption less revenue elastic.

These first experiments show that households in the income distribution are heterogeneously affected by price change. The next step will be to define an additional fiscal policy instrument to reduce the adverse effect of the increase in the price of carbon at the bottom of the distribution.

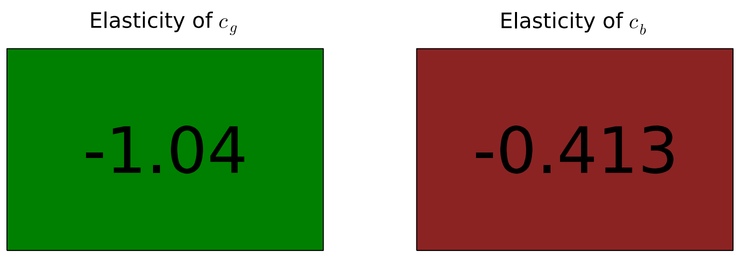


FIGURE 6. DECREASING THE WAGE BY 5%.

| Parameter | Description | Value |
|---|---|---|
| | Production | |
| $\alpha_G$ | cap. share Green | .33 |
| $\alpha_B$ | cap. share Bown | .33 |
| $\delta$ | cap.depreciation | 0.1 |
| $\alpha_K$ | El. sub cap. sector | 0.1 |
| | Preference and Income | |
| $\sigma$ | Utility Function Curvature | 2.0 |
| $\bar{c}$ | Brown Minimal Consumption | 0.033 |
| $\omega_{G,\theta}$ | Green Consumption Utility Weight | 0.96 |
| $\omega_{B,\theta}$ | Brown Consumption Utility Weight | 0.04 |
| $\alpha_\theta$ | CES Substitution Parameter | -0.06 |
| $\rho$ | Income Shock Persistence | 0.96 |
| $\epsilon$ | Income Shock Std. Dev. | 0.10 |
| $\tau_G^c$ | Tax on Green Consumption | 0.0 |
| $\tau_B^c$ | Tax on Browbn Consumption | 0.0 |
| $T$ | Government Transfer | 0.0 |
| | Model outcome | |
| $r$ | Interest rate | 0.028 |
| $w$ | Wage | .400 |
| $p_G$ | Pre-tax Price of Green Good | 0.48 |
| $p_B$ | Pre-tax Price of Brown Good | 0.52 |
| | Climate parameters | |
| $\gamma_s$ | Damage function parameter | 5.3e-5 |
| $\bar{S}$ | Pre-ind. stock of atm. emissions | 581 GtC |
| $S_0$ | Current (2021) stock of atm. emissions | 845 GtC |
| $\delta_m$ | Emissions decay parameter | 0.0006 |
| $m$ | Emission int. brown sector | 0.459e-12 tC |

Table 1—the table

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## Appendices

Full model specification

$$
\begin{aligned}
P_t^{mod} &= p_{G,t}\left(1+\left(\frac{\omega_{G,\theta}}{\omega_{B,\theta}}\right)^{\frac{1}{\alpha_\theta-1}}\left(\frac{p_{B,t}}{p_{G,t}}\right)^{\frac{\alpha_\theta}{\alpha_\theta-1}}\right)\\
T_t^{mod} &\equiv T-p_{B,t}\left(\bar{c}_{B,\theta}-\left(\frac{p_{B,t}\omega_{G,\theta}}{p_{G,t}\omega_{B,\theta}}\right)^{\frac{1}{\alpha_\theta-1}}\bar{c}_{G,\theta}\right)
\end{aligned}
$$

- Unconstrained agents

$$
\begin{aligned}
\left(c_{G,t}-\bar{c}_{G,\theta}\right)^{-\sigma} &= \beta\mathbb{E}\left(1+r_{t+1}\right)\left(c_{G,t+1}-\bar{c}_{G,\theta}\right)^{-\sigma}\\
P_t^{mod}c_{G,t}+a_{t+1} &= (1+r_t)a_t+w_t y+T_t^{mod},
\end{aligned}
$$

- Constrained agents

$$
\begin{aligned}
a_{t+1} &= \bar{a}\\
P^{mod}c_{G,t} &= (1+r_t)a_t+w_t y+T_t^{mod},
\end{aligned}
$$

$$\int a\,d\Lambda_t = K_t, \tag{A1}$$

$$\int c_B\left(a\right)d\Lambda_t = C_{B,t} \tag{A2}$$

$$\int c_G\left(a\right)d\Lambda_t = C_{G,t} \tag{A3}$$

$$C_{G,t}+I_{G,K,t} = Y_{G,t}, \tag{A4}$$

$$C_{B,t}+I_{B,K,t} = Y_{B,t}, \tag{A5}$$

$$T_t = \tau^c_{G,t}\tilde{p}_{G,t}C_{G,t}+\tilde{p}_{B,t}\tau^c_{B,t}C_{B,t}$$

$$\bar{K}_{t-1} = K_{B,t} + K_{G,t}$$

$$\bar{L} = L_{B,t} + L_{G,t},$$

$$C_{G,t} + I_{G,t} = Y_{G,t},$$

$$C_{B,t} + I_{B,t} = Y_{B,t}.$$

$$Y_{B,t} = A_{B,t} K_{B,t}^{\alpha_B} L_{B,t}^{1-\alpha_B}$$

$$Y_{G,t} = A_{G,t} K_{G,t}^{\alpha_G} L_{G,t}^{1-\alpha_G}$$

$$\tilde{r}_t = \alpha_G \tilde{p}_{G,t} A_{G,t} K_{G,t}^{\alpha_G - 1} L_{G,t}^{1-\alpha_G} - \delta = \alpha_B \tilde{p}_{B,t} A_{B,t} K_{B,t}^{\alpha_s - 1} L_{B,t}^{1-\alpha_s} - \delta$$

$$\tilde{w}_t = (1-\alpha_G) \tilde{p}_{G,t} A_{G,t} K_{G,t}^{\alpha_s} L_{G,t}^{-\alpha_G} = (1-\alpha_B) \tilde{p}_{B,t} A_{B,t} K_{B,t}^{\alpha_s} L_{B,t}^{-\alpha_B}$$

$$\begin{aligned} I_t &= \left(\omega_{G,K} I_{G,t}^{\alpha_K} + \omega_{B,K} I_{B,t}^{\alpha_K}\right)^{\frac{1}{\alpha_K}} \\ \bar{K}_t &= I_t + (1-\delta)\,\bar{K}_{t-1} \end{aligned}$$

$$I_{G,t} = \left(\frac{\tilde{p}_{G,t}}{\omega_{G,K}}\right)^{\frac{1}{\alpha_K - 1}} I_t \text{ and } I_{B,t} = \left(\frac{\tilde{p}_{B,t}}{\omega_{B,K}}\right)^{\frac{1}{\alpha_K - 1}} I_t$$

$$1 = \omega_{G,K}^{\frac{1}{1-\alpha_K}} \tilde{p}_{G,t}^{\frac{\alpha_K}{\alpha_K - 1}} + \omega_{B,K}^{\frac{1}{1-\alpha_K}} \tilde{p}_{B,t}^{\frac{\alpha_K}{\alpha_K - 1}}$$

$$\begin{aligned} S_t &= m Y_{B,t-1} + S_{t-1}(1-d_m). \\ A_{B,t} &= A_{0,B} A_t \left(1 - D_s(S_t)\right) \\ A_{G,t} &= A_{0,G} A_t \left(1 - D_s(S_t)\right) \\ 1 - D_s(S_t) &= e^{-\gamma_s (S_t - \overline{S})}, \end{aligned}$$