

Application from Erich French (2005)

ScPo Graduate Labor

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Application to Labor Supply: French (2005)

- How does labor supply relate to the **Social Security** and **Pension systems**?
- How does health enter this picture?
- We need a dynamic model here: you will work differently if you expect to get a pension at age 60.

French (2005) Structure

- People in the model can save (not borrow).
- There is random income each period.
- People probabilistically age (i.e. if they don't die, they get a year older)
- Health evolves over this lifecycle in a state-dependent fashion.

Policy Simulations

- Once estimated, use the model for policy experiments
- Change retirement age from 62 to 63.
- Reduce Social Security (SS) Benefits by 20 %
- Eliminate tax wage from SS meanstest.

Model

- Choose consumption and hours C_t, H_t as well as whether to apply for SS, $B_t \in \{0, 1\}$

- Health status is M_t , which enters utility:

$$U(C_t, H_t, M_t)$$

- There is a **bequest function** $b(A_T)$, where A_t are assets.
- Denote s_t the probability of being alive at t , given alive at $t - 1$.
- Hence, $S(j, t) = \frac{1}{s_t} \prod_{k=t}^j s_k$ is prob of living through j , given alive at t
- You die for sure in $T + 1$, hence $s_{T+1} = 0$.

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Model

- At time t , lifetime utility is

$$\mathbb{E}_t \left[\sum_{j=t+1}^{T+1} \beta^j S(j-1, t) (s_j U(C_j, H_j, M_j) + (1-s_j)b(A_j)) \right] + U(C_t, H_t, M_t) \quad (1)$$

- Applying for SS sets $B_t = 1$
- The **problem of the consumer** is to choose a plan $\{C_j, H_j, B_j\}_{t=j}^{T+1}$ that maximizes (1), subject to
 - 1 mortality determination
 - 2 health determination
 - 3 wage determination
 - 4 spousal income
 - 5 and a budget constraint.

Utility Function

Within period utility is

$$U(C_t, H_t, M_t) = \frac{1}{1-\nu} \left(C_t^\gamma (L - H_t - \theta_P P_t - \phi \mathbf{1}\{M = \text{bad}\})^{1-\gamma} \right)^{1-\nu} \quad (2)$$

where

- 1 leisure is $L - H_t - \theta_P P_t - \phi \mathbf{1}\{M = \text{bad}\}$
- 2 Health is good/bad when $M = \{0, 1\}$.
- 3 If participate in labor force, $P_t = 1$, and pay fixed cost θ_P
- 4 Retirement: set $H = 0$. Can re-enter labor at any time.

Role of Utility

The distribution of annual hours of work is clustered around both 2000 and 0 hours of work, a regularity in the data that standard utility functions have a difficult time replicating. Fixed costs of work are a common way of explaining this regularity in the data (Cogan, 1981). Fixed costs of work generate a reservation wage for a given marginal utility of wealth. Below the reservation wage, hours worked is zero. Slightly above the reservation wage, hours worked may be large. Individual level labour supply is highly responsive around this reservation wage level although wage increases above the reservation wage result in a smaller labour supply response.

Bequest, Health and Survival

- The bequest function is

$$b(A_t) = \theta_B \frac{(A_t + K)^{(1-\nu)\gamma}}{1 - \nu}$$

- Survival is a function of age and health

$$s_{t+1} = s(M_t, t + 1)$$

- Health evolves according to a markov chain

$$\pi_{\text{good,bad},t+1} = \Pr(M_{t+1} = \text{good} | M_t = \text{bad}, t + 1)$$

Wages

- Wages are given by

$$\ln W_t = \alpha \ln H_t + W(M_t, t) + AR_t \quad (3)$$

- $W(\cdot)$ is an age profile, AR an autoregressive component.

$$AR_t = \rho AR_{t-1} + \eta_t, \eta \sim N(0, \sigma_\eta^2)$$

- Spousal income is

$$ys_t = ys(W_t, t) \quad (4)$$

Budget constraint

The budget constraint is

$$A_{t+1} = A_t + Y(x_t, \tau) + B_t \times ss_t - C_t \geq 0 \quad (5)$$

with the function Y measuring post tax income, r interest, pb pension, ss social security, and

$$x_t = rA_t + W_t H_t + y s_t + p b_t + \epsilon_t \quad (6)$$

- Social Security available only after age 62. It lasts till death.
- Similarly, pension benefits are paid out after age 62.

Social Security and Retiring at 65

- SS benefits depend on Averaged Indexed Monthly Earnings, **AIME**. This is average earnings in the 35 highest-paying years.
- Incentives to start SS at age 65: from 62-65, every early year reduces benefits by 6.7%. From 65-70, every additional work year increases benefits only by 3%.
- Under 70 year-olds who work while on SS are taxed heavily. Earnings above 6000\$ per year are taxed at 50% (plus federal, state and payroll tax).

Pensions

- Pensions are similar. Pension wealth is typically illiquid before a certain age.
- Also a function of AIME.
- Focus on **defined benefit pension** plans. The pension formula is *defined and known* in advance. The pension does not depend on stock market returns, for example.

Model Solution

- The state vector is $X_t = (A_t, W_t, B_t, M_t, AIME_t)$
- Preferences are $\theta = (\gamma, \nu, \theta_P, \theta_B, \phi, L, \beta)$
- The value function solves

$$\begin{aligned} V_t(X_t) = & \max_{C_t, H_t, B_t} \{ U(C_t, H_t, M_t) \\ & + \beta s_{t+1} \sum_{M \in \text{good, bad}} \int V_{t+1}(X_{t+1}) dF(W_{t+1} | M_{t+1}, W_t, t) \pi(M_{t+1} | M_t, t) \\ & + \beta(1 - s_{t+1} b(A_{t+1})) \} \end{aligned} \quad (7)$$

- Backward induction over grids \mathcal{X} and \mathcal{C}, \mathcal{H}

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Estimation: 2 Steps

Estimation proceeds in two steps:

- 1 Estimate laws of motion for exogenous state variables outside of the model, and calibrate others to reasonable values.
- 2 Use those processes as given in the model, and find a set of preference parameters that generates model output as close as possible to observed data.

Simulated Method of Moments as Extremum Estimator

Definition: Extremum Estimator

An estimator $\hat{\theta}$ of dimension p is called an **extremum estimator** if there is a scalar objective function $Q_n : \mathbb{R}^p \mapsto \mathbb{R}$ and a parameter space $\Theta \subset \mathbb{R}^p$ such that

$$\hat{\theta} = \arg \max_{\theta \in \Theta} Q_n(\theta)$$

- OLS
- GMM/SMM
- IV
- MLE

GMM moment function

The **Generalized Method of Moments** relies on a set of K moment conditions which provide limited information about a parameter θ , given a set of data w_i :

$$\mathbb{E}[g(w_i; \underbrace{\theta}_{p \times 1})] = \underbrace{\mathbf{0}}_{K \times 1} \quad (8)$$

This has **sample analog**:

$$g_n(\theta) = \frac{1}{n} \sum_{i=1}^n g(w_i; \theta) \quad (9)$$

Can we solve (8)?

Definition: GMM Estimator

The **GMM estimator** $\hat{\theta}(\hat{W})$ of θ satisfying

$$\mathbb{E}[g(w_i; \underbrace{\theta}_{p \times 1})] = \underbrace{\mathbf{0}}_{K \times 1}$$

with $\theta \in \Delta \subset \mathbb{R}^p$ is defined as

$$\hat{\theta}(\hat{W}) = \arg \min_{\theta \in \Delta} n g_n(\theta)' \hat{W} g_n(\theta) \quad (10)$$

where \hat{W} is a $K \times K$ weighting matrix that may depend on the data and is symmetric and p.d.

GMM \rightarrow SMM

- **Simulated Method of Moments** uses simulated data $\tilde{w}_i(\theta)$ instead of w_i .
- Notice the main difference: the (simulated!) data now depend on the model parameters θ .
- The problem is still to choose θ such that (9) is as close to zero as possible. We only modify (9) slightly to read

$$g_n(\theta) = \frac{1}{n} \sum_{i=1}^n g(\tilde{w}_i(\theta)) \quad (11)$$

- Making this change, the estimator is still defined by (10)

SMM Practicalities

- Notice that obtaining a sample of simulated data $\{\tilde{w}_i(\theta)\}_{i=1}^N$ involves
 - 1 Solving the dynamic program (7), $V_t(X_t, \theta)$
 - 2 Using the optimal policies, simulating a panel of data.

where step 1 usually is costly in terms of computation time.

- The precise form of the moment function g is, to some extent, at the researcher's discretion.
- **Which moments to include in g** is a similar question to **which variables to include in my regression**.

French (2005): Moment Function g

The moments the model must match are:

- 1 Median assets
- 2 Mean assets
- 3 Mean participation conditional on health
- 4 Mean hours conditional on health

See Equations (14)-(17) in the paper for how those are computed.

Estimating Lifecycle Profiles from Data

- If t indexes calendar time and i individuals, define a generic **lifecycle profile** of variable x by

$$\mathbb{E}[x_{it} | \text{age}_i] \quad (12)$$

- Written as in (12), this contains individual fixed effects, year effects and family size effects. A complicated object to interpret.
- Probably better would be

$$\mathbb{E}[x_{it} | \text{age}_i, t, i, M_{it}, \text{famsize}_{it}]$$

Question: How is this conditional expectation identified?

Estimating Lifecycle Profiles from Data

- To address those issues, French proposes

$$Z_{it} = f_i + \sum_{m \in \text{good, bad}} \sum_{k=1}^T \Pi_{mk} \mathbf{1}[\text{age}_{it} = k | m] \\ + \sum_{j=1}^J \mathbf{1}[\text{famsize}_{it} = j] + \Pi_U U_t + u_{it}$$

where Z stands for assets, hours, participation or wages.

- The required age profiles are the sequences of parameters $\sum_{k=1}^T \Pi_{mk}$.

Selected Wage Data

- A problem with this is that wage data is only observed for workers.
- **Classical Question:** What about the wage of non-workers? This is really like in Heckman (1977).
- If wage growth were the same for workers and non-workers, this would not be a problem – the fixed effect estimator identifies wage growth.
- French (2005) proposes to compare selection bias in real and simulated data in an iterative procedure.

Data

- Panel Study of Income Dynamics (PSID) 1968–1997.
- Rich income data, assets in several waves, labor supply, and also health data are included.
- PSID is a great dataset to work with.
- <https://cran.r-project.org/web/packages/psidR/index.html> is a great package to build it.

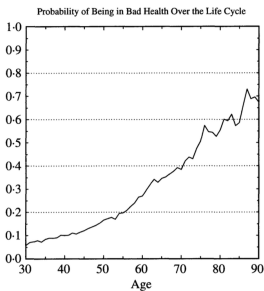
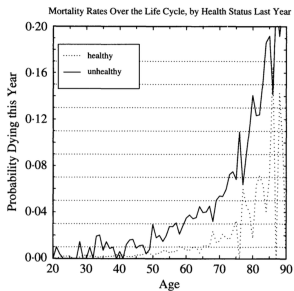
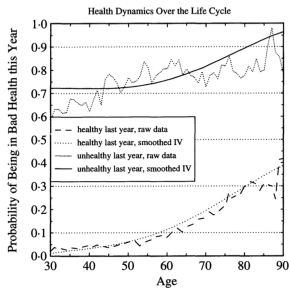
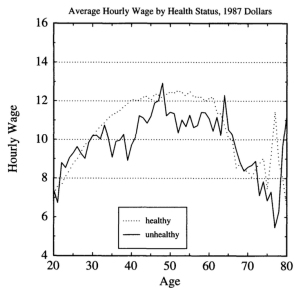
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Model

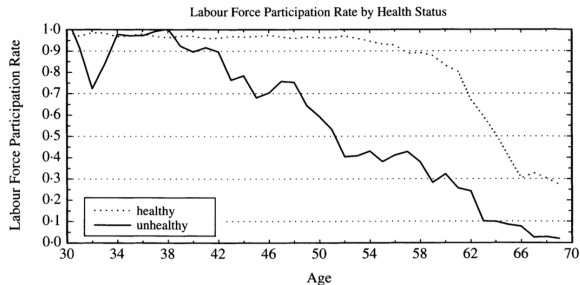
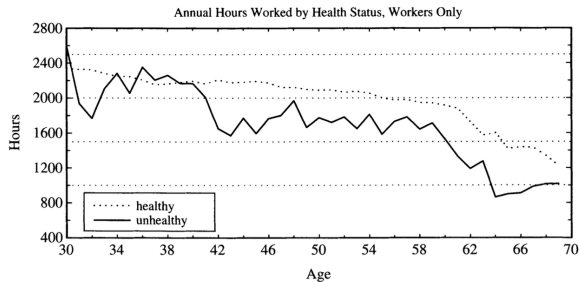
Estimation

Results

Results: Exogenous profiles 1



Results: Exogenous profiles 2



Results: Parameter Estimates

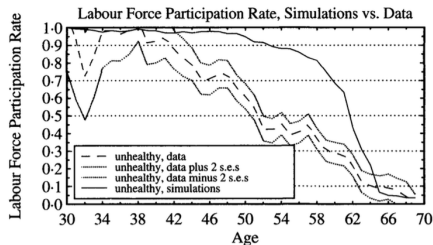
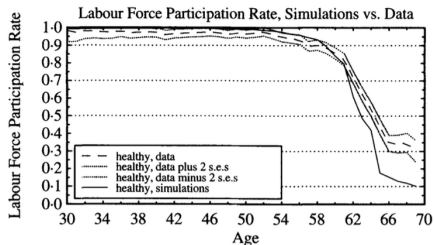
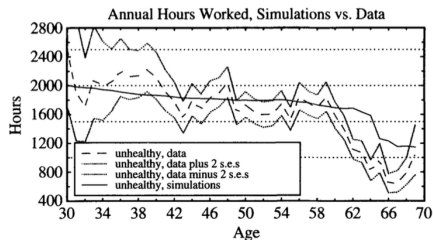
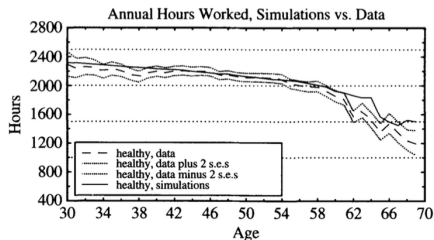
Parameter and definition	Specification			
	(1)	(2)	(3)	(4)
γ Consumption weight	0.578 (0.003)	0.602 (0.003)	0.533 (0.003)	0.615 (0.004)
ν Coefficient of relative risk aversion, utility	3.34 (0.07)	3.78 (0.07)	3.19 (0.05)	7.69 (0.15)
β Time discount factor	0.992 (0.002)	0.985 (0.002)	0.981 (0.001)	1.04 (0.004)
L Leisure endowment	4466 (30)	4889 (32)	3900 (24)	3399 (28)
ϕ Hours of leisure lost, bad health	318 (9)	191 (7)	196 (8)	202 (6)
θ_P Fixed cost of work, in hours	1313 (14)	1292 (15)	335 (7)	240 (6)
θ_B Bequest weight	1.69 (0.05)	2.58 (0.07)	1.70 (0.04)	0.037 (0.001)
χ^2 Statistic: (233 degrees of freedom)	856	880	830	1036
$\epsilon_{h,w}(40)$ Labour supply elasticity, age 40	0.37	0.37	0.35	0.19
$\epsilon_{h,w}(60)$ Labour supply elasticity, age 60	1.24	1.33	1.10	1.04
Reservation hours level, age 62	885	916	1072	1051
Coefficient of relative risk aversion	2.35	2.68	2.17	5.11

Standard errors in parentheses

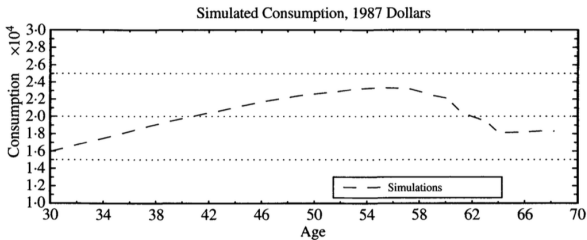
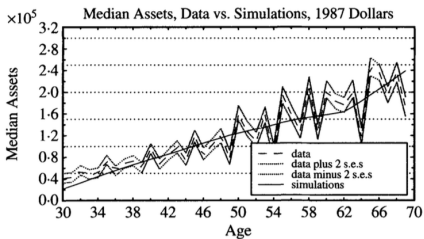
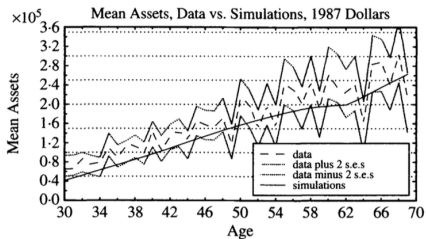
Specifications described below:

- (1) Does not account for selection or tied wage-hours offers
- (2) Accounts for selection but not tied wage-hours offers
- (3) Accounts for tied wage-hours offers but not selection
- (4) Accounts for selection and tied wage-hours offers

Results: Model Fit 1



Results: Model Fit 2



Experiments

Baseline (1987 policy environment) vs

- 1 Redues SS benefits by 20%
- 2 Increase early retirement age from 62 to 63.
- 3 Eliminate SS earnings test for over 65 year olds.
- 4 Earnings test was in fact abolished in 2000 - look at model predictions.

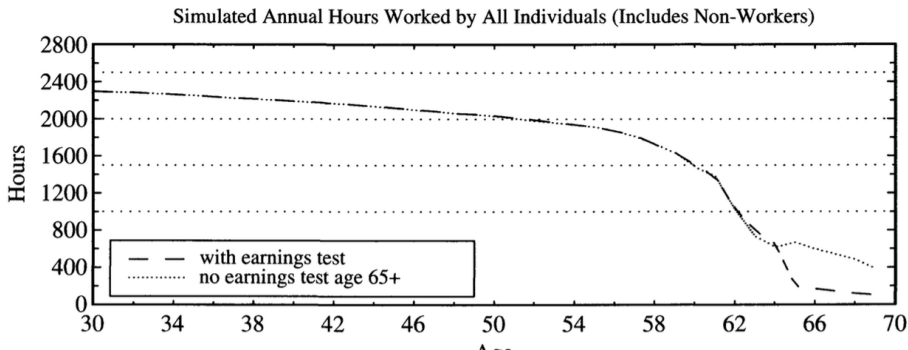
Results: Policy Experiments

	Years worked	Hours worked per year	PDV of labour income (\$)	PDV of consumption (\$)	Assets at age 62 (\$)
With borrowing constraints					
1987 policies	32-60	2097	1781	1583	190
Reduce benefits	action after 62	2099	1789	1569	200
Reduce benefits, reduce taxes	33-00	2115	1803	1586	203
Shift early retirement age to 63	32-62	2096	1781	1584	190
Eliminate earnings test, age 65+	33-62	2085	1799	1594	188
Without borrowing constraints					
1987 policies	32-39	2067	1764	1603	158
Reduce benefits	32-58	2063	1770	1587	168
Reduce benefits, reduce taxes	32-68	2078	1781	1602	170
Shift early retirement age to 63	32-39	2067	1764	1603	158
Eliminate earnings test, age 65+	33-46	2063	1784	1616	154

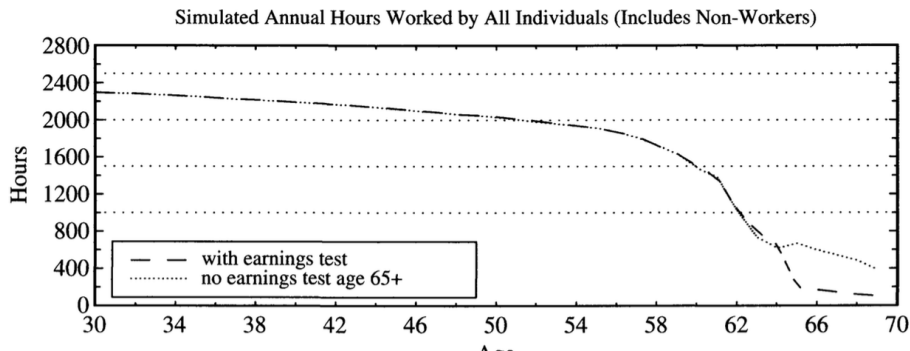
PDV stands for present discounted value.

Consumption, labour income, and assets are measured in thousands.

Results: Model Fit 2



Results: Model Fit 2



- Removing earnings test increases lifetime wealth.
- Leisure being normal good, this increases demand for C as well as L
- Hence participate for more years.

Conclusions

- French (2005) constructs an estimable lifecycle model with labour supply, retirement, savings and social security.
- People would spend 3 months more in work if SS were decreased by 30%.
- The sharp drop in participation at age 65 is explained by actuarial unfairness of pension and SS around that age.

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James J Heckman. Sample selection bias as a specification error (with an application to the estimation of labor supply functions), 1977.