# Graduate Labor Economics 

## Wage Dispersion

Why are similar workers paid differently?

Based on Mortensen (2003)<br>Florian Oswald, SciencesPo Paris

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## Classical Labor Demand

- Assume a representative Firm, each worker produces $y$.
- $L$ is amount of labor, fraction $q$ of jobs destroyed each instant.
- Assume convex adjustment cost of labor $C(\cdot)$.
- In steady state, $L$ constant and firm replaces $q L$ workers each instant.


## Profit:

$$
\Pi=L y-[w L+C(q L)]
$$

FOC:

$$
\begin{equation*}
y=q C^{\prime}(q L)+w \tag{1}
\end{equation*}
$$

Equation (1) defines labor demand.

## Classical Equilibrium

## Labor Supply:

- Assume $N$ workers, each with reservation wage $z(c d f H(z))$.
- $i$ works only if $w_{i}>z_{i}$, hence labor supply is $N H(w)$.


## Equilibrium:

$$
\begin{align*}
y & =q C^{\prime}\left(q N H\left(w^{*}\right)\right)+w^{*}  \tag{2}\\
L^{*} & =\operatorname{NH}\left(w^{*}\right)
\end{align*}
$$

(1) No unemployment (people prefer non-participation at $w^{*}$ ).
(2) Efficient (decentralized solution coincides with planner's).
(3) There is a unique wage $w^{*}$.
(4) Segementing firms into $J$ classes yields $J$ wages $w_{j}^{*}$.

## Classical Equilibrium



## Figure 9.20

The competitive equilibrium.

## Classical Equilibrium

- Note that the wage is given to market participants via the equilibrium.
- The firm cannot choose the wage.
- The firm could choose a lower wage than $w^{*}$ : nobody would work for them.
- The firm could choose a higher wage than $w^{*}$ : everybody would work for them, but in the long run they would make a loss (since pay $w$ in excess of marginal product of labor!)


## Introduction

- This law of one price does not hold in labor market data.
- The standard human capital wage equation explains about $30 \%$ of wage variation.
- What are the remaining $70 \%$ ?
- Worker Unobservables? $\Rightarrow$ Worker fixed effect.
- Differential Wage Policies? $\Rightarrow$ Firm fixed effect.
- Sizeable literature after Abowd et al. (1999): Both!
- But what's the theory behind different wage policies?
- Let's start with some evidence.


## Evidence for Wage Dispersion

- Oi and Idson (1999) reported huge variation in mean wages.
- Working in large firm increases male mean wage by 70\%.
- Some industries pay much better than others.
- This holds even after accounting for observable worker characteristics.


## Evidence for Wage Dispersion

- Oi and Idson (1999) reported huge variation in mean wages.
- Working in large firm increases male mean wage by 70\%.
- Some industries pay much better than others.
- This holds even after accounting for observable worker characteristics.
- Note: $R^{2}$ is $40 \%$ for men and $35 \%$ for women. There is still lots of dispersion to be explained!


## Oi and Idson (1999)

Table 1.1
Average hourly earnings (in USD) by industry, sex, and firm size (May 1983 CPS)

|  | In firms with an employment of |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Industry and sex | No. of workers | $1-24$ | $1000+$ | Ratio |
| Male |  |  |  |  |
| Agriculture | 4,667 | 4.388 | 6.436 | 1.467 |
| Mining | 12,369 | 8.316 | 13.487 | 1.622 |
| Construction | 9,380 | 7.995 | 13.679 | 1.711 |
| Manufacturing | 10,300 | 7.344 | 11.705 | 1.594 |
| Trans./comm. | 11,541 | 7.761 | 13.096 | 1.687 |
| Trade | 7,433 | 6.253 | 8.438 | 1.349 |
| Finance | 11,696 | 8.437 | 12.588 | 1.492 |
| Services | 8,677 | 7.526 | 10.020 | 1.331 |
| Women |  |  |  |  |
| Agriculture | 4,696 | 4.556 | 5.013 | 1.100 |
| Mining | 9,606 | 9.917 | 9.706 | 0.979 |
| Construction | 6,687 | 6.344 | 8.262 | 1.302 |
| Manufacturing | 6,880 | 6.032 | 7.714 | 1.279 |
| Trans./comm. | 8,697 | 5.722 | 9.787 | 1.710 |
| Trade | 4,858 | 4.403 | 5.269 | 1.197 |
| Finance | 6,902 | 6.193 | 7.538 | 1.217 |
| Services | 6,656 | 5.955 | 7.759 | 1.303 |
| Source: Oi and Idson (1999), Table 6. |  |  |  |  |

## Oi and Idson (1999)

Table 1.2
Wage equation coefficients by sex, May 1983 CPS $^{\text {a }}$

| Variable | Male employees |  |  | Female employees |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | $\beta$ | $t$-value | Mean | $\beta$ | $t$-value |
| Firm/plant |  |  |  |  |  |  |
| Size dummies ${ }^{\text {b }}$ |  |  |  |  |  |  |
| F2SP | 0.030 | 0.110 | 3.96 | 0.032 | 0.088 | 3.06 |
| F3SP | 0.025 | 0.092 | 3.04 | 0.27 | 0.127 | 4.06 |
| F4SP | 0.008 | 0.147 | 2.76 | 0.007 | 0.048 | 0.83 |
| F5SP | 0.051 | 0.117 | 5.17 | 0.040 | 0.131 | 4.96 |
| F2LP | 0.115 | 0.087 | 5.32 | 0.116 | 0.075 | 4.41 |
| F3LP | 0.109 | 0.142 | 8.38 | 0.124 | 0.127 | 7.50 |
| F4LP | 0.043 | 0.134 | 5.53 | 0.055 | 0.160 | 7.00 |
| F5LP | 0.353 | 0.245 | 17.90 | 0.316 | 0.232 | 17.00 |
| Industry |  |  |  |  |  |  |
| Agriculture | 0.025 | -0.351 | -11.28 | 0.005 | -0.170 | -2.40 |
| Mining | 0.024 | 0.193 | 6.31 | 0.005 | 0.326 | 4.69 |
| Construction | 0.084 | 0.186 | 9.91 | 0.012 | 0.079 | 1.70 |
| Trans./comm. | 0.094 | 0.103 | 6.08 | 0.055 | 0.161 | 6.86 |
| Trade | 0.216 | -0.129 | $-9.53$ | 0.240 | -0.190 | -12.44 |
| Finance | 0.055 | 0.031 | 1.43 | 0.119 | -0.006 | -0.35 |
| Service | 0.162 | -0.112 | -7.49 | 0.350 | -0.026 | -1.84 |
| Statistics |  |  |  |  |  |  |
| $R^{2}$ | 0.4064 |  |  | 0.3352 |  |  |
| N | 7,833 |  |  | 5,973 |  |  |

$$
\begin{aligned}
& \ln \left(y_{i t}\right)=\beta x_{i t}+u_{i t} \\
& \text { where } x \text { contains } \\
& \text { - education } \\
& \text { - experience \& } \\
& \text { job tenure } \\
& \text { - marital status } \\
& \text { - race } \\
& \text { - location }
\end{aligned}
$$

## Source: Oi and Idson (1999), Table 9.

${ }^{\text {a }}$ Dependent variable is $\ln$ (hourly earnings).
${ }^{\text {b }}$ F2-F5 correspond to firm size categories 25-99, 100-499, 500-999, 1000+; SP, LP corre-
spond to small plants ( $1-24$ ) and large plants ( $25+$ ), respectively.

## Davis and Haltiwanger (1996) - Firm size differentials

MEAN WAGES AND WAGE DISPERSION BY PLANT SIZE (PROD WORKERS)

$\rightarrow$ MEAN $\quad-$ COEFF OF VAR

MEAN WAGES AND WAGE DISPERSION BY PLANT SIZE(NONPROD WORKERS)

$\rightarrow$ MEAN $\quad+$ COEFF OF VAR

## Abowd et al. (1999) - Firm-Worker Match

Abowd and coauthors in a series of papers consider versions of the linear statistical model

$$
y_{i t}=\alpha_{i}+\gamma_{J(i, t)}+x_{i t} \beta+\varepsilon_{i t}
$$

where

- $y_{i t}$ : worker $i$ 's compensation (demeaned)
- $\alpha_{i}$ : worker $i$ 's fixed effect
- $\gamma_{j}$ : firm $j$ 's fixed effect
- $J(i, t)=k$ : worker $i$ 's firm at time $t$ is $k$.
- $x_{i t}$ : worker's time varying characteristics (demeaned)


## Abowd et al. (1999) - Results

- Abowd, Creecy, Kramarz (2002): roughly 50\% of wage differences across industries is unobserved worker ability, the rest is difference in firm's wage policies.
- Both worker and firm fixed effects matter.
- Interestingly, they find a close to zero correlation between both coefficients, implying no sorting.
- This is hotly debated, we will dedicate more time to this in another session.


## Why do Firms have different wage policies?

- The sole idea of a firm-specific wage policy implies at least some
market power
of firms: how else could they set their own wage?
- So, not the perfectly competitive labour market described above.
- Pure monopsony? A single buyer of labor.
- Firms might face upward sloping labor supply curve, given competitors' wages.
- The more I pay, the more applicants I'll get, given others' wages.


## Paul Samuelson (1951, p. 554)

The fact that a firm of any size must have a wage policy is additional evidence of labor market imperfections. In a perfectly competitive market a firm need not make decisions on its pay schedules; instead it twould turn to the morning newpaper to learn what its wage policy would have to be. Any firm, by raising wages ever so little, could get all the extra help it wanted. If, on the other hand, it cut the wage ever so little, it would find no labor to hire at all. [...]

Availability of labor does, therefore, affect the wage you set under realistic conditions of imperfect competition. If you are a very small firm you may even bargain and haggle with prospective workers so as to not pay more than you have to. But if you are of any size at all, you will name a wage for each type of job, then decide how many of the applicants will be taken on.

## Martin Bronfenbrenner (1956, pp 577-578)

The typical employer in an unorganized labor market is by no means a pure competitior facing market wages which he cannot alter. The mobility of the labor force, even between firms located close together, is low by reason of the inability of workers to wait for employment or risk unemployment, plus the inadequacy of the information usually available to them regarding alternative employment opportunities. This low mobility permits each to set his own rates and form his own labour market within limits which are sometimes quite wide. In the technical jargon of economic theory, the typical employer in an unorganized labor market has some degree of monopsony power and can set his own wage policy.

## Frictions.

- So it seems there are some frictions.
- Workers don't know the wages offered by all firms: There is imperfect information.
- Firms realize that the labor supply curve they face is not perfectly elastic.
- If the law of one price where to hold, we wouldn't see similar workers being paid differently.


## Classical Labor Demand

## Beyond Neoclassical Labor

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## Theories

What do we want from a theory?

- A notion of monopsony.
- Labor supply curves that are upward sloping in own wage.


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- Labor supply curves that are upward sloping in own wage.


## Some Candidates:

(1) Compensating Differentials
(2) Efficiency Wages
(3) Sorting
(4) Search Theories

- Different reservation wages of workers
- Outcome of wage setting game of firms
- Bilateral bargaining


## Theories

What's needed?

- A notion of monopsony.
- Labor supply curves that are upward sloping in own wage.


## Some Candidates:

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## Pure Wage Dispersion

## Setup:

(1) There are $m$ identical firms and $n$ identical workers.
(2) There is just a single time period.
(3) All workers are unemployed.
(4) Friction: Workers are completely uninformed about wages at beginning of period.
(5) Firms can only inform a subset of workers: they send offers to random workers.
(6) Each worker applies for highest offer in their sample of offers.
(7) Firms realize that workers may get more than one offer. Set wage that takes this into account!

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(7) Firms realize that workers may get more than one offer. Set wage that takes this into account!
Surprise: The unique equilibrium to this game will generate a unique non-degenerate distribution of offers. Even when all employers are identical!

## Setup for a Non-cooperative Game between Firms

- CRS production: Marginal revenue of labor is constant $p$.
- Workers prefer higher wage, but work only for $w \geq b$
- Firm only employs if $p \geq w$
- To prevent Bertrand result $w=p$, assume each firm can contact only 1 worker.
- Firm can compute probability that worker got multiple offers.

Two-stage game:
(1) All firms choose wage and send offer to random worker.
(2) Each worker accepts best offer at hand.

Key object for firm: Probability that worker $i$ accepts any given offer.

## How many offers do workers receive?

Let's start with how many offers a worker can expect to get:

- Let $X_{i}$ be total number of offers received by $i$
- $X \sim \operatorname{Binomial}\left(m, \frac{1}{n}\right)$
- for large $n, m$, Binomial $\left(m, \frac{1}{n}\right) \approx \operatorname{Poisson}\left(\frac{m}{n}\right)$
- We get

$$
\begin{equation*}
\operatorname{Pr}\{X=x\}=\frac{e^{-\lambda} \lambda^{x}}{x!}, \quad \text { where } \lambda=\frac{m}{n} \tag{3}
\end{equation*}
$$

which is called the contact frequency.

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Will show: The Prob. of accepting any offer will depend only on $\lambda$ and the rank of offer $w$, given by $\operatorname{cdf} F(w)$

## WTS: No symmetric pure strategy equilibrium

- Expected profit:

$$
\begin{equation*}
\pi(p, w, F(w))=P(F(w), \lambda)(p-w) \tag{4}
\end{equation*}
$$

where Probability of Acceptance: $P(F(w), \lambda)$
No Pure Strategies: Profitable deviation from any masspoint.

- Suppose all other firms offer $w<p$. Deviating to $w^{\prime}=w+\varepsilon$ brings more profit with certainty, because worker accepts for sure. (Deviating like this is always profitable. [show])


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- Suppose all other firms offer $w<p$. Deviating to $w^{\prime}=w+\varepsilon$ brings more profit with certainty, because worker accepts for sure. (Deviating like this is always profitable. [show])
- If all offer $w=p$, no deviation to $w^{\prime}=w+\varepsilon$, since that generates a loss. However, deviating to $w=b$ yields expected pos profit, since $\operatorname{Pr}\{X=0\}=e^{-\lambda}>0$
- I.e. workers accept $b$ iff it is their only offer.
- There is no mass point of firms at $b$.


## Characterizing the equilibrium offer distribution

- Only workers with a single offer accept the lowest wage offered.
- There is no mass point of firms offering the same wage (see previous slide).
- Hence, the lower bound of the offer support is the reservation wage $b$.

$$
\underline{w}=\arg \max _{w \geq b} \pi(p, w, 0)=\arg \max _{w \geq b} e^{-\lambda}(p-w)=b
$$

## Characterizing the equilibrium offer distribution

- The lower bound of wage support is $\underline{w}=b$.
- The upper bound of wage support is $\bar{w}=p$. (Why?)
- All offers must generate same expected profit in equilibrium.
- There can be no gap in the offer cdf. [Draw.]


## Characterizing the equilibrium offer distribution

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- All offers must generate same expected profit in equilibrium.
- There can be no gap in the offer cdf. [Draw.]

This yields

## Proposition 1.

Any equilibrium distribution offer $F(w)$ is continuous and has connected support on $[b, p]$. Hence, we have $F:[b, p] \mapsto[0,1]$.

## Probability of Acceptance $P(F(w), \lambda)$

- Probability that my offer $w$ exceeds $x$ other offers you received is $F(w)^{x}$.
- $x$ is Poisson as characterized in (3)
- Hence the probability of accepting my offer is [show]

$$
\begin{align*}
P(F(w), \lambda) & =\sum_{x=0}^{\infty} F(w)^{x} \frac{e^{-\lambda} \lambda^{x}}{x!} \\
& =e^{-\lambda[1-F(w)]} \tag{5}
\end{align*}
$$

- increasing in $w$, decreasing in $\lambda$.


## Equilibrium Offer Distribution

- Tradeoff: Higher offer (lower profit) vs higher acceptance
- Profit must be equal for any $w$.
- Hence,

$$
\begin{aligned}
\pi(p, w, F(w)) & =(p-w) e^{-\lambda[1-F(w)]}= \\
\pi(p, w, 0) & =(p-w) e^{-\lambda} \quad \forall w \in[b, \bar{w}]
\end{aligned}
$$

- Which implies [show]

$$
\begin{align*}
F(w) & =\frac{1}{\lambda} \ln \left(\frac{p-b}{p-w}\right)  \tag{6}\\
\bar{w} & =p-\frac{p+b}{e^{-\lambda}} \tag{7}
\end{align*}
$$

- Comparative statics as $\lambda \rightarrow \infty$ ?


## Extension: Different Wages across Industies

- We have seen big pay differences between industries.
- Can this simple model generate a positive correlation between labor productivity and wages?


## Need to show:

Optimal wage of high productivity firm $p^{\prime}$ is greater or equal to the one of low type $p, p^{\prime}>p$.

## Extension: Different Wages across Industies

- Expected profit per contacted worker:

$$
\begin{equation*}
\pi(p, w, F(w))=P(F(w), \lambda)(p-w) \tag{8}
\end{equation*}
$$

- Optimal wage choice for $p$ type:

$$
\begin{equation*}
w(p)=\arg \max _{w \geq b} \pi(p, w, F(w)) \tag{9}
\end{equation*}
$$

- Resulting optimal profit is

$$
\begin{align*}
\pi^{*}(p) & =\max _{w \geq b} \pi(p, w, F(w)) \\
& =\max _{w \geq b} e^{-\lambda[1-F(w)]}(p-w) \tag{10}
\end{align*}
$$

## Extension: Different Wages across Industies

## Proposition 2.

Given any two firms, the more productive offers a higher wage and expects a strictly greater profit per worker contacted. That is, $p^{\prime \prime}>p^{\prime} \Longrightarrow \pi^{*}\left(p^{\prime \prime}\right)>\pi^{*}\left(p^{\prime}\right)$ and $w^{\prime \prime} \geq w^{\prime}$, for all $w^{\prime \prime} \in$ $w\left(p^{\prime \prime}\right)$ and $w^{\prime} \in w\left(p^{\prime}\right)$.

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Proof. Given $p^{\prime \prime}>p^{\prime}$ and $w^{\prime \prime} \in w\left(p^{\prime \prime}\right), w^{\prime} \in w\left(p^{\prime}\right)$

$$
\begin{align*}
\pi^{*}\left(p^{\prime \prime}\right) & =P\left(F\left(w^{\prime \prime}\right), \lambda\right)\left(p^{\prime \prime}-w^{\prime \prime}\right) \\
& \geq P\left(F\left(w^{\prime}\right), \lambda\right)\left(p^{\prime \prime}-w^{\prime}\right) \\
& >P\left(F\left(w^{\prime}\right), \lambda\right)\left(p^{\prime}-w^{\prime}\right)=\pi^{*}\left(p^{\prime}\right) \\
& \geq P\left(F\left(w^{\prime \prime}\right), \lambda\right)\left(p^{\prime}-w^{\prime \prime}\right) \tag{11}
\end{align*}
$$

## Corollary to Proposition 2

## Corollary to Proposition 2.

i More productive firm makes more profit. (Strict inequality)
ii $w^{\prime \prime} \geq w^{\prime}$ follows from

$$
\left(p^{\prime \prime}-p^{\prime}\right) P\left(F\left(w^{\prime \prime}\right), \lambda\right) \geq\left(p^{\prime \prime}-p^{\prime}\right) P\left(F\left(w^{\prime}\right), \lambda\right)>0
$$

and because $P\left(F\left(w^{\prime}\right), \lambda\right)$ strictly increasing, see (5)

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$$

and because $P\left(F\left(w^{\prime}\right), \lambda\right)$ strictly increasing, see (5)

## Implications:

(1) Any possible $w^{*}\left(p^{\prime \prime}\right)$ is no smaller than any $w^{*}\left(p^{\prime}\right)$
(2) But this implies that $w$-supports of types $p^{\prime \prime}$ and $p^{\prime}$ intersect only in a single point, i.e. the boundary.
(3) By previous argument (no mass points), $\bar{w}^{\prime}=\underline{w}^{\prime \prime}$.

## Example with two types $p^{\prime \prime}>p^{\prime}$

- Consider $p^{\prime \prime}>p^{\prime}$. We have $\underline{w}_{1}=b$ and $\bar{w}_{1}=\underline{w}_{2}$.
- Profit must be equal among firms of same type:

$$
\begin{aligned}
\pi^{*}\left(p^{\prime}\right) & =P(F(b), \lambda)\left(p_{1}-b\right) \\
& =P(F(w), \lambda)\left(p_{1}-w\right), \forall w \in\left[b, \bar{w}_{1}\right], \text { and } \\
\pi^{*}\left(p^{\prime \prime}\right) & =P\left(F\left(\bar{w}_{1}\right), \lambda\right)\left(p_{2}-\underline{w}_{2}\right) \\
& =P(F(w), \lambda)\left(p_{2}-w\right), \forall w \in\left[\underline{w}_{2}, \bar{w}_{2}\right]
\end{aligned}
$$

- This implies an equilibrium distribution:

$$
F(w)= \begin{cases}\frac{1}{\lambda} \ln \left(\frac{p_{1}-b}{p 1-w}\right) & \text { for } w \in w\left(p_{1}\right)=\left[b, \bar{w}_{1}\right]  \tag{12}\\ \frac{1}{\lambda} \ln \left(\frac{p_{2}-w_{2}}{p 2-w}\right) & \text { for } w \in w\left(p_{2}\right)=\left[\underline{w}_{2}, \bar{w}_{2}\right]\end{cases}
$$

- Hence: Offers and productivity are positively correlated.
- Hence: Microfoundation for employer fixed effect in regression.


## Extension: Firm Size Differentials

- High type $p^{\prime \prime}$ has incentive to attract more workers than low type.
- Introduce recruiting effort to generate $\operatorname{cov}(w, n(p))>0$ where $n(p)$ is firm size.
- Let $v$ be the number of workers contacted, $c(v)$ convex cost.
- Expected profit: $\pi(p, w, F(w)) v-c(v)$
- Need to optimally choose both wage $w$ and effort $v$ now.


## Extension: Firm Size Differentials

The firms problem is to

$$
\max _{w \geq b, v \geq 0}\{\pi(p, w, F(w)) v-c(v)\}
$$

with first order condition [show]

$$
\begin{equation*}
c^{\prime}(v(p))=\pi^{*}(p) \tag{13}
\end{equation*}
$$

Take away: Given $p^{\prime \prime}>p^{\prime}$,
(1) $\Longrightarrow v\left(p^{\prime \prime}\right)>v\left(p^{\prime}\right)$
(2) Expected size $n(p)=P(F(w(p)), \lambda) v(p), n\left(p^{\prime \prime}\right)>n\left(p^{\prime}\right)$
(3) $w(p)$ positively, not perfectly, correlated with $n(p)$

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The Burdett-Mortensen Model

## Burdett and Mortensen (1998)

- We now extend this setup to more periods.
- Again firms post different wages but make the same expected profit.
- Unemployed workers now have the option to wait for better offers.
- We allow job-to-job transitions: workers may quit for a better job.
- We assume a stationary wage contract.
- Time runs forever and in discrete steps of size $\Delta$.


## Burdett and Mortensen (1998) - Acceptance probability

- Firms still contact workers at random.
- Workers are unemployed with probability equal to unemployment rate $u$.
- Workers accept offers greater than reservation wage $R$ with probability

$$
\begin{align*}
P(F(w), \Delta \lambda) & =\sum_{x=0}^{\infty} F(w)^{x} \frac{e^{-\Delta \lambda}(\Delta \lambda)^{x}}{x!} \\
& =e^{-\lambda \Delta[1-F(w)]} \tag{14}
\end{align*}
$$

where now $\Delta \lambda$ is the average num of offers received in timespan $\Delta$.

- Worker accepts another offer $w^{\prime}$ iff $w^{\prime}>w$.


## Probability of getting a worker with offer $w$

- Considering pool of available workers and their individivual acceptance probs, the firm gets a worker when posting wage $w$ with probability

$$
\begin{equation*}
h(w)=[\underbrace{u}_{\text {Unemployed }}+\underbrace{(1-u) G(w)}_{\text {Employed at less than } w}] \cdot P(F(w), \Delta \lambda) \tag{15}
\end{equation*}
$$

- Expected profit for the firm is

$$
\pi(p, w, F(w))=h(w) J(p, w)
$$

where $J$ is the expected present value of future profits.

- $J$ has to take in to account possibility that worker quits.


## Deriving the EPV of the Firm: $J$

- Probability of staying $=$ All $x$ other offers are less than $w$, i.e. $F(w)^{x}$
- Number of offers received is Poisson as before, hence the quit probability is

$$
\begin{align*}
Q(F(w), \lambda \Delta) & =\sum_{x=0}^{\infty}\left[1-F(w)^{x}\right] \frac{e^{-\Delta \lambda}(\Delta \lambda)^{x}}{x!} \\
& =1-P(F(w), \Delta \lambda) \tag{16}
\end{align*}
$$

i.e. prob. of quitting is equal to prob that current offer is not highest offer.

## Deriving the EPV of the Firm: $J$

- Instantaneuos profit is $p-w$.
- Hence, firms earn that for as long as the worker stays. In present value terms,

$$
(1+r \Delta) J(p, w)=\overbrace{(p-w) \Delta}^{\text {profit during } \Delta}+[1-\delta \Delta-\underbrace{Q(F(w), \lambda \Delta)}_{P(\text { quits })}] J(P, w)
$$

where $\delta$ is the exogenous job destruction rate.

- let's rearrange this to

$$
\begin{equation*}
J(p, w)=\frac{(p-w)}{r+\delta+Q(F(w), \lambda \Delta) / \Delta} \tag{18}
\end{equation*}
$$

## Deriving the EPV of the Firm: $J$

- Burdett and Mortensen (1998) assume sequential search in continuous time.
- Then, $h(w)$ and $J(p, w)$ are the limit of earlier expression as $\Delta \rightarrow 0$ :

$$
\begin{align*}
h(w) & =\lim _{\Delta \rightarrow 0}[u+(1-u) G(w)] \cdot P(F(w), \Delta \lambda) \\
& =u+(1-u) G(w)  \tag{19}\\
\text { and } &  \tag{20}\\
J(p, w) & =\lim _{\Delta \rightarrow 0} \frac{(p-w)}{r+\delta+Q(F(w), \lambda \Delta) / \Delta} \\
& =\frac{(p-w)}{r+\delta+\lambda[1-F(w)]} \tag{21}
\end{align*}
$$

## Optimal wage choice

- We now have simple expression for expected profit $\pi(p, w)$ :

$$
\begin{equation*}
\pi(p, w)=h(w) J(p, w)=\frac{(u+(1-u) G(w))(p-w)}{r+\delta+\lambda[1-F(w)]} \tag{22}
\end{equation*}
$$

- With higher wage firm trades off longer retention of worker vs smaller flow profits.
- Firms take others' offer $F(w)$ and wage distributions $G(w)$ as given.


## Reservation Wage

- Denoting $W(w)$ and $U$ the values of work and unemployment, the reservation wage is such that $W(R)=U$.

$$
\begin{align*}
r U & \left.=b+\lambda\left[\int \max (U, W(x)) d F(x)\right)-U\right]  \tag{23}\\
r W(w) & \left.=w+\lambda\left[\int \max (W(w), W(x))\right)-W(w)\right] d F(x) \\
& +\delta[U-W(w)] \tag{24}
\end{align*}
$$

- The reservation wage $R$ solves $W(R)=U$. Set equal and evaluate at $R$ to arrive at

$$
R=b
$$

- Notice the simplification that the arrival rate $\lambda$ is the same for employed/unemployed.


## Flow Equations

- All offers are above or equal to $R=b$ in eqm. Hence, unemployed exit at rate $\lambda$ from unemployment.
- Unemployment evolves as:

$$
\begin{equation*}
\dot{u}=\delta(1-u)-\lambda u \tag{25}
\end{equation*}
$$

- Denote $E(w)=(1-u) G(w)$ the fraction of workers employed at $W \leq w$. This evolves as

$$
\begin{equation*}
\dot{E}(w)=\underbrace{\lambda F(w) u}_{\text {unemp. accept } W \leq w}-\underbrace{(\delta+\lambda[1-F(w)])}_{\text {unemp+higher } w} \underbrace{E(w)}_{\text {curr. } W \leq w} \tag{26}
\end{equation*}
$$

## Flows in Equilibrium

- In equilibrium (25) and (26) are both equal to zero.
- In other words, every solution to this system converges to

$$
\begin{align*}
\frac{u}{1-u} & =\frac{\delta}{\lambda}  \tag{27}\\
G(w) \equiv \frac{E(w)}{1-u} & =\frac{\delta F(w)}{\delta+\lambda[1-F(w)]} \tag{28}
\end{align*}
$$

## Equilibrium Definition

## Steady-state Equilibrium

A Steady-state Equilibrium solution is given by a contact frequency $\lambda$, an offer $\operatorname{cdf} F(w)$, an employment rate $u$, and a cdf of wages $G(w)$, such that

- $F(w), u$ and $G(w)$ satisfy (27) and (28),
- every offer maximizes expected profit per worker and
- there is free entry for firms.


## Equilibrium Existence and Uniqueness

- As in the single period model: all offers must yield same expected profit.
- Hence, only eqm $F(w)$ is cdf with connected support $[b, \infty]$
- Equal profit (sub (28) and (27) into (22)) shows existence and uniqueness:

$$
\begin{align*}
\pi(p, b) & =\frac{\delta}{\delta+\lambda} \frac{p-b}{r+\delta+\lambda}=  \tag{29}\\
\pi(p, w) & =\left(\frac{\delta}{\delta+\lambda[1-F(w)]}\right)\left(\frac{p-w}{r+\delta+\lambda[1-F(w)]}\right) \tag{30}
\end{align*}
$$

- Notice that (29) is constant in $w$, while (30) is
(1) increasing in $F(w)$
(2) decreasing in $w$
which implies a unique intersection of both, hence a unique equilibrium.


## Free Entry

- To close model, assume recruiting cost is constant $c$.
- Firms make zero expected profit per worker.
- In other words,

$$
\begin{equation*}
\pi(p, b)=c \tag{31}
\end{equation*}
$$

- You can solve this equation to get the associated equilibrium objects $F, G, u$.


## Conclusion

- We studied a simple version of Burdett and Mortensen (1998) in a one-shot and continuous time environment.
- Workers search while on the job, firms send random offers to workers.
- There is no firm or worker heterogeneity in this simple model.
- Firms post permanent contracts.
- Workers employ rejection wage strategy.


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